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Thermally induced vibrations of piezo-thermo-viscoelastic composite beam with relaxation times and system response

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Abstract

Purpose – The purpose of this paper is to present numerical studies on thermally induced vibrations of piezo-thermo-viscoelastic composite beam subjected to a transient thermal load using coupled finite element method.

Design/methodology/approach – The thermal relaxation and viscoelastic relaxations are taken into consideration to obtain the system response. The concept of "memory load" along with the thermal relaxation is accounted for viscoelastic core material. The influence of type of core material on the response of the system also analyzed.

Findings – The findings show viscoelastic behavior with relaxation times in composite sandwich structures.

Originality/value – The paper shows accounting relaxation times as a memory load in composite sandwich structures.

Keywords Composite materials, Relaxation theory, Vibration, Thermal properties of materials, Piezoelectricity

Paper type Research paper

1. Introduction

The development of intelligent composite materials with piezoelectric components under thermal environment offers great potential for their use in advanced aerospace structural applications due to high strength to weight and high stiffness to weight ratios. Aircraft and space vehicles usually work in severe temperature environments and the heat sources change quite often from time to time. A significant variation of temperature in a solid may cause deformation due to thermal expansion or contraction. Depending on the constraint, the solid can bend, elongate or subjected to thermal stresses. If the temperature varies rapidly, vibration may also occur, which can affect the dynamics and stability of the structures. Therefore, thermally induced vibration is an important concern for the design of these structures.

Thermal induced vibrations are basically vibrations generated in structures subjected to transient thermal loading. When these structures subjected to thermal shock loading, finiteness speed of propagation of heat front becomes important.

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In Fourier heat diffusion, it is implicitly assumed that speed of propagation of heat is infinite. The finiteness speed of propagation of heat can be modeled by introducing two thermal relaxation times in the coupled constitutive equations (Green and Lindsay, 1972) or by using the modified Fourier law of heat conduction (Lord and Shulman, 1967) with one relaxation time. These theories are known as generalized thermo-elastic theories. In the present study, the constitutive model proposed by Green and Lindsay (1972) is used. The present work deals with the thermally induced vibration of composite beam treated with piezo and viscoelastic layers subjected to transient thermal load.

The classical Fourier law of heat conduction leads to instantaneous propagation of heat to infinitely remote areas of space. This paradox is traditionally surmounted by describing the heat signal as a wave, called second sound. One of the most popular generalizations of the thermoelasticity theory allowing for the second sound is the model of Green and Lindsay (GL) (1972). Prohofsky and Krumhansl (1964) discuss the feasibility of experimental observation of thermal pulses in dielectric materials, indicating the optimum temperature and frequency range for the observation of second sound.

The application of generalized thermo-elastic theories to piezoelectric media using finite element method is an emerging research field. For example, Mindlin (1974) presented the high-frequency vibrations of thermo-piezoelectric crystal plates. Tiahanu *et al.* (2002, 2004) studied the generalized thermal shock problem of a thick piezoelectric plate based on Green and Lindsay (G-L) theory. They extended research to half space in electro-magneto-thermo-elasticity using Lord and Shulman (L-S) theory. Recently, Tran *et al.* (2007) studied the thermally induced vibration and its control for thin isotropic and laminated composite plates using finite element method.

In many applications of piezo-electrically controlled smart structures, the functionality of the system has to be ensured even in an extremely hot and cold condition. Hence, the thermal effect is very important and must be taken into account when designing such structures. Gornandt and Gabbert (2002) have investigated the static and dynamic response of thermo-piezoelectric smart structures using finite element analysis by considering combined thermal, electric and mechanical excitations. Srinatha and Lewis (1981) presented the stress analysis of plane problems in linear thermo-viscoelasticity using finite element formulation. Bargmann (1974) discussed memory effects in the mechanical and thermal response, i.e. viscoelasticity and second sound. Johnson and Tessler (1995) investigated the viscoelastic internal variable constitutive theory applicable to higher order elastic beam theory using finite element formulation. The behavior of the viscous material is approximately modeled as a Maxwell solid. Rao and Sunar (1993) derived thermo-piezoelectric beam finite element formulation to model the distributed actuation and sensing in a thermal environment. Raja *et al.* (2004) investigated the thermally induced vibration control for composite plates and shells with piezoelectric active damping. Muki and Sternberg (1961) presented the quasi-static analysis of transient thermal stresses in the linear theory of viscoelastic solids with temperature dependent properties. The modeling is based on "thermorheologically simple" material model.

Tzou and Howard (1994) discussed the piezo-thermo-elastic thin shell theory applied to active structures. Tzou and Ye (1994) presented the piezo-thermo-elastic effects of distributed piezoelectric sensor/actuator and structural systems. Pradeep (2006) investigated the piezothermo-viscoelastic isotropic beams by considering the two thermal relaxation times and viscoelastic relaxation function. Lee and Saravanos (1996) analyzed the complete coupled mechanical, electrical, and thermal response of piezoelectric composite beams by accounting the thermal effects. Kapuria *et al.* (2004) developed the zig-zag theory for the static and dynamic electro-thermo-mechanical analysis of piezoelectric layer composite beams.

Manolis and Beskos (1980) developed a general numerical method for determining the dynamic response of beam structures to rapidly applied thermal loads. Yi *et al.* (1999) developed finite element algorithm for the efficient analysis of smart composite structures with piezoelectric layer based on piezo-electro-hygro-thermo-visco-elastsicity. Song *et al.* (2003) presented the transient disturbance in a half space under thermoelasticity with two relaxation times due to moving internal heat source using G-L theory. Sherief (1993, 1994) presented the state space formulation for thermo-elasticity with two relaxation times. He extended the study to thermo-mechanical shock problems using half space governing equations. Pitman and Ni (1994) modeled the phase transition by a viscoelastic relaxation law, and described the numerical experiments on this system. Arup *et al.* (2004) investigated the two-dimensional problems of thermo-elasticity dealing with thermo-elastic wave propagation in a rotating medium with relaxation effects. Using the joint Laplace and Fourier transforms.

From the literature, it is found that there is no research has been done on the piezo-thermo-visco-elastic composite beam by considering the two thermal relaxation times and viscoelastic relaxation function and corresponding temperature shift function. The present study shows realistic behavior of the viscoelastic material under time domain analysis by taking into account "memory load" of previous time step. The effect of temperature on the material response function is accounted by assuming the temperature-time equivalence hypothesis and recasting the equations in terms of shifted time.

2. Finite element formulation

Finite element formulation is derived for piezo-thermo-viscoelastic composite beam by combining the finite element equation for piezo-thermo-elastic problem given by Tiahanu *et al.* (2002) and thermo-viscoelastic formulation presented by Johnson and Tessler (1995) and Srinatha and Lewis (1981). The eight-node plane stress element is developed for the analysis. The temperature and memory (time) effects of viscoelastic material are incorporated as a load vector in the solution procedure. Newmark-β method is employed to solve the coupled dynamic equations. Piezo-thermo-elastic and thermo-viscoelastic formulations are given in the following sections.

2.1 Piezo-thermo-elastic formulation

The constitutive equations for a generalized piezo-thermo-elastic material can be written as (Tiahanu *et al.*, 2002):

$$
\begin{gathered}
\{\sigma\} = [C]\{\varepsilon\} - [e]\{E\} - \{a\}(\theta + \tau_1\dot{\theta}) \\
\{D\} = [e]^T\{\varepsilon\} + [p]\{E\} - \{d\}(\theta + \tau_1\dot{\theta}) \\
\rho\eta = \{a\}^T\{\varepsilon\} + \{d\}^T\{E\} - c(\theta + \tau_2\dot{\theta}) \quad \{q\} = -[k]\{\theta'\}
\end{gathered}
\tag{1}
$$

Here, $\{\sigma\}$, $\{D\}$, $\{q\}$ and η are stress, electric displacement, heat, flux, and entropy density, respectively. $\{\varepsilon\}, \{E\}, \theta, \dot{\theta}$ and $\{\theta'\}$ are strain, electric field, temperature difference, rate of change of temperature, and temperature gradient, respectively. $[C], [e], [a], [p], [d]$ and $[k]$ are stiffness coefficients, piezo electric coupling coefficients, stress temperature coefficients, dielectric constants, pyroelectric coupling, and thermal conductivity matrices, respectively. For plane stress problem plane stress reduced constitutive relation are used. $C = \rho C_E/T_0$, here ρ is the density, C_E is the specific heat and T_0 is reference temperature. τ_1 and τ_2 are thermal relaxation times. In these set of constitutive equation (1) first equation is the stress equation, second is electric displacement equation, third and fourth put together will give the heat conduction equation. The introduction of relaxation times is due to a rigorous thermodynamic analysis by Green and Lindsay (1972). The terms multiplied with relaxation times indicate the finiteness of the propagation of that particular quantity. When τ_1 and τ_2 are set to zero the constitutive relations reduce to conventional coupled piezo-thermo-elastic constitutive relations. Eight-noded iso-parametric plane stress element is developed using the above constitutive model.

Arrays of elemental degrees of freedom (d.o.f) are:

$$\begin{aligned} \{u_e\} &= \{u_1 v_1 u_2 v_2 \ldots u_8 v_8\}^T && \text{Mechanical d.o.f} \\ \{\phi_e\} &= \{\phi_1 \phi_2 \ldots \phi_8\}^T && \text{Electrical d.o.f} \\ \{\theta_e\} &= \{\theta_1\, \theta_2 \ldots \theta_8\}^T && \text{Thermal d.o.f} \end{aligned} \tag{2}$$

The displacements ($\{u_e\} = \{uv\}$), electric potential (ϕ) and temperature difference (θ) within the element can be expressed in terms of shape functions and corresponding nodal quantities as follows:

$$\{u\} = [N_u]\{u_e\}, \quad \phi = [N_\phi]\{\phi_e\}, \quad \theta = [N_\theta]\{\theta_e\} \tag{3}$$

$$\begin{aligned} [N_u] &= \begin{bmatrix} N_1 & 0 & N_2 & 0 & \ldots & N_8 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & \ldots & N_8 \end{bmatrix}, \\ \{N_\phi\} &= \{N_\theta\} = \left\{ N_1 \;\; N_2 \;\; \ldots \;\; N_8 \right\} \end{aligned} \tag{4}$$

$N_1, N_2, \ldots, N_8$ are the shape functions. The strain-displacement relation, electric fields and temperature gradients for a plane stress two-dimensional piezo-thermo-elastic case can be written as:

$$\begin{aligned} \{\varepsilon\} &= \left\{ \frac{\partial u}{\partial x} \; \frac{\partial v}{\partial y} \; \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\}^T && \text{Mechanical strains} \\ \{E\} &= \left\{ -\frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial y} \right\}^T && \text{Electric fields} \\ \{\theta'\} &= \left\{ \frac{\partial \theta}{\partial x} \; \frac{\partial \theta}{\partial y} \right\}^T && \text{Temperature gradients} \end{aligned} \tag{5}$$

The above equations can be expressed in terms of derivatives of shape functions as follows:

$$\{\varepsilon\} = [B_1]\{u_e\}, \quad E = -[B_2]\{\phi_e\}, \quad \theta' = [B_2]\{\theta_e\} \tag{6}$$

where $[B_1]$ and $[B_2]$ are derivatives of shape function related to mechanical strains and electric fields, temperature gradient, respectively, are expressed as follows:

$$[B_1] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \cdots & \frac{\partial N_8}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & \cdots & 0 & \frac{\partial N_8}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_8}{\partial y} & \frac{\partial N_8}{\partial x} \end{bmatrix}, \tag{7}$$

$$[B_2] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_8}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_8}{\partial y} \end{bmatrix}$$

The variational form of equation (6) is:

$$\delta\{\varepsilon\} = [B_1]\delta\{u_e\}, \quad \delta\{E\} = -[B_2]\delta\{\phi_e\}, \quad \delta\{\theta'\} = [B_2]\delta\{\theta_e\} \tag{8}$$

Considering the body force $\{f\}$, the virtual displacement principle for piezo-thermo-elastic problem can be written in the following form:

$$\begin{aligned} &\int_V (\delta\{\varepsilon\}^T\{\sigma\} - \delta\{E\}^T\{D\} - \delta\{\theta'\}^T\{q\} - \delta\theta\rho T_0\{\dot{\eta}\})dV \\ &= \int_V \delta\{u\}^T(\{f\} - \rho\{\ddot{u}\})dV + \int_{A_\sigma} \delta\{u\}^T\{\bar{T}\}dA + \int_{A_w} \delta\phi\bar{w}dA + \int_{A_q} \delta\theta\bar{q}dA \end{aligned} \tag{9}$$

The L.H.S of the equation (9) shows the virtual work statement represents the internal virtual work, and RHS shows the work done by external loads. $\{f\}$ is the body force, $\rho\{\ddot{u}\}$ are the inertia forces. A_σ, A_w and A_q are the surface areas on which traction $\{\bar{T}\}$, electric charge $\bar{w}$ and heat flux $\bar{q}$ are imposed. δ is the variational operator. Substituting the constitutive equation (1) into equation (9) by using the equations (3) and (6) the each term of virtual work expression are expressed as follows:

$$\begin{aligned} \int_V \delta\{\varepsilon\}^T\{\sigma\}dV &= \delta\{u^e\}\int_V [B_1]^T([C][B_1]\{u^e\} + [e]([B_2]\{\phi^e\}) \\ &\quad - \{a\}\left[N_\theta^e\right]^T(\{\theta^e + \tau_1\dot{\theta}^e\}))dV \\ &= \delta\{u^e\}^T\left(\left[K_{uu}^e\right]\{u^e\} + \left[K_{u\phi}^e\right]\{\phi^e\} - \left[K_{u\theta}^e\right]\{\theta^e\} - [C_{u\theta}]\{\dot{\theta}^e\}\right) \end{aligned} \tag{10}$$

$$\int_V (-\delta\{E\}^T\{D\})dV = \delta\{\phi^e\}^T \int_V [B_2]^T([e]^T[B_1][u^e] + [p](-[B_2]\{\phi^e\}) + [d]\left[N_\theta^e\right]^T(\{\theta^e + \tau_1\dot{\theta}^e\}))dV$$
$$= \delta\{\phi^e\}^T\left(\left[K_{\phi u}^e\right]\{u^e\} - \left[K_{\phi\phi}^e\right]\{\phi^e\} + \left[K_{\phi\theta}^e\right]\{\theta^e\} + [C_{\phi\theta}]\{\dot{\theta}^e\}\right) \tag{11}$$

$$\int_V \delta\{\theta'\}^T\{q\}dV = \delta\{\theta^e\}^T \int_V [B_2]^T(-[k][B_2]\{\theta^e\})dV = -\delta\{\theta^e\}^T\left[K_{\theta\theta}^e\right]\{\theta^e\} \tag{12}$$

$$\int_V (-\delta\theta\rho T_0\{\dot{\eta}\})dV = -\delta\{\theta^e\}^T \int_V (T_0[N_\theta^e])\{a\}^T[B_1]\{\dot{u}^e\} - [d]^T([B_2]\{\dot{\phi}^e\}) + c\left[N_2^e\right]^T\{\dot{\theta}^e\} + c\tau_2\left[N_\theta^e\right]^T\{\ddot{\theta}^e\})dV$$
$$= \delta\{\theta^e\}^T\left(-\left[C_{\theta u}^e\right]\{\dot{u}^e\} + \left[C_{\theta\phi}^e\right]\{\dot{\phi}^e\} - \left[C_{\theta\theta}^e\right]\{\dot{\theta}^e\} - \left[M_{\theta\theta}^e\right]\{\ddot{\theta}^e\}\right) \tag{13}$$

$$\int_V \delta\{u\}^T(\{f\} - \rho\{\ddot{u}\})dV = \delta\{u^e\}^T \int_V \left[N_u^e\right]^T(\{f\} - \rho\left[N_u^e\right]\{\ddot{u}^e\})dV$$
$$= \delta\{u^e\}^T\left(\{f_m^e\} - \left[M_{uu}^e\right]\{\ddot{u}^e\}\right) \tag{14}$$

$$\int_{A_\sigma} \delta\{u\}^T\{\bar{T}\}dA = \delta\{u^e\}^T \int_{A_\sigma} \left[N_u^e\right]\{\bar{T}\}dA = \delta\{u^e\}^T\left\{T_m^e\right\} \tag{15}$$

$$\int_{A_w} \delta\phi\{\bar{w}\}dA = \delta\{\phi^e\}^T \int_{A_w} \left[N_\phi^e\right]\{\bar{w}\}dA = \delta\{\phi^e\}^T\left\{T_e^e\right\} \tag{16}$$

$$\int_{A_q} \delta\theta\{\bar{q}\}dA = \delta\{\theta^e\}^T \int_{A_q} \left[N_\theta^e\right]\{\bar{q}\}dA = \delta\{\theta^e\}^T\left\{T_\theta^e\right\} \tag{17}$$

Substituting the equations (10)-(17) in the virtual work expression (9) and equating the like terms the following finite element equation can be arrived:

$$\left[M_{uu}^e\right]\{\ddot{u}^e\} - \left[C_{u\theta}^e\right]\{\dot{\theta}^e\} + \left[K_{uu}^e\right]\{u^e\} + \left[K_{u\phi}^e\right]\{\phi^e\} - \left[K_{u\theta}^e\right]\{\theta^e\} = \{f_m^e\} + \left\{T_m^e\right\} \tag{18}$$

$$-\left[C_{\phi\theta}^e\right]\{\dot{\theta}^e\} - \left[K_{\phi u}^e\right]\{u^e\} + \left[K_{\phi\phi}^e\right]\{\phi^e\} - \left[K_{\phi\theta}^e\right]\{\theta^e\} = -\left\{T_e^e\right\} \tag{19}$$

$$\left[M_{\theta\theta}^e\right]\{\ddot{\theta}^e\} + \left[C_{\theta u}^e\right]\{\dot{u}^e\} - \left[C_{\theta\phi}^e\right]\{\dot{\phi}^e\} + \left[C_{\theta\theta}^e\right]\{\dot{\theta}^e\} + \left[K_{\theta\theta}^e\right]\{\theta^e\} = -\{T_\theta^e\} \tag{20}$$

equations (18)-(20) can be expressed in matrix form as follows:

$$\begin{bmatrix} M_{uu}^e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_{\theta\theta}^e \end{bmatrix} \begin{Bmatrix} \ddot{u}^e \\ \ddot{\phi}^e \\ \ddot{\theta}^e \end{Bmatrix} + \begin{bmatrix} 0 & 0 & -C_{u\theta}^e \\ 0 & 0 & -C_{\phi\theta}^e \\ C_{\theta u}^e & -C_{\theta\phi}^e & C_{\theta\theta}^e \end{bmatrix} \begin{Bmatrix} \dot{u}^e \\ \dot{\phi}^e \\ \dot{\theta}^e \end{Bmatrix}$$
$$+ \begin{bmatrix} K_{uu}^e & K_{u\phi}^e & -K_{u\theta}^e \\ -K_{\phi u}^e & K_{\phi\phi}^e & -K_{\phi\theta}^e \\ 0 & 0 & K_{\theta\theta}^e \end{bmatrix} \begin{Bmatrix} u^e \\ \phi^e \\ \theta^e \end{Bmatrix} = \begin{Bmatrix} f_m^e + T_m^e \\ -T_e^e \\ -T_\theta^e \end{Bmatrix} \tag{21}$$

The elemental matrices used in the equation (21) are defined as:

$$[K_{uu}^e] = \int_V [B_1]^T [C][B_1] dV, \quad \left[K_{u\phi}^e\right] = \int_V [B_1]^T [e][B_2] dV$$

$$[K_{u\theta}^e] = \int_V [B_1]^T [a][N_\theta]^T dV, \quad \left[K_{\phi\phi}^e\right] = \int_V [B_2]^T [p][B_2] dV$$

$$\left[K_{\phi\theta}^e\right] = \int_V [B_2]^T [d][N_\theta]^T dV, \quad [K_{\theta\theta}^e] = \int_V [B_2]^T [k][B_2] dV$$

$$\left[K_{\phi u}^e\right] = \int_V [B_2]^T [e]^T [B_1] dV, \quad [C_{\theta\theta}^e] = \int_V T_0 [N_\theta] c [d]^T [N_\theta]^T dV$$

$$[C_{\theta u}^e] = \int_V T_0 [N_\theta][a]^T [B_1] dV, \quad [C_{u\theta}^e] = \int_V \tau_1 [B_1]^T [a][N_\theta]^T dV = \tau_1 [K_{u\theta}]$$

$$\left[C_{\theta\phi}^e\right] = \int_V T_0 [N_\theta][d]^T [B_2] dV, \quad \left[C_{\phi\theta}^e\right] = \int_V \tau_1 [B_2]^T [d][N_\theta]^T dV = \tau_1 \left[K_{\phi\theta}^e\right]$$

$$[M_{uu}^e] = \int_V [N_u]^T \rho [N_u] dV, \quad [M_{\theta\theta}^e] = \int_V T_0 [N_\theta] c \tau_2 [N_\theta]^T dV = \tau_2 [C_{\theta\theta}^e]$$

$$\{T_e^e\} = \int_{A_w} \left[N_\phi^e\right] \{\bar{w}\} dA, \quad \{T_\theta^e\} = \int_{A_q} [N_\theta^e] \bar{q} dA$$

$$\{f_m^e\} = \int_V [N_u^e]^T \{f\} dV, \quad \{T_m^e\} = \int_{A_\sigma} [N_u^e]^T \{\bar{T}\} dA$$

The equation (19) can be rewritten by assuming there is no external electrical loading as:

$$\{\phi^e\} = \left[K_{\phi\phi}^e\right]^{-1} \left(\left[K_{\phi u}^e\right] \{u^e\} + \left[C_{\phi\theta}^e\right] \{\dot{\theta}^e\} + \left[K_{\phi\theta}^e\right] \{\theta^e\} \right) \tag{22}$$

Substituting the value of $\{\phi^e\}$ in the equations (18) and (20) can be written as:

$$
\begin{aligned}
&[M_{uu}^{e}]\{\ddot{u}^{e}\} - [C_{u\theta}^{e}]\{\dot{\theta}^{e}\} + [K_{uu}^{e}]\{u^{e}\} \\
&\quad + \left[K_{u\phi}^{e}\right]\left(\left[K_{\phi\phi}^{e}\right]^{-1}\left(\left[K_{\phi u}^{e}\right]\{u^{e}\} + \left[C_{\phi\theta}^{e}\right]\{\dot{\theta}^{e}\} + \left[K_{\phi\theta}^{e}\right]\{\theta^{e}\}\right)\right) \\
&\quad - [K_{u\theta}^{e}]\{\theta^{e}\} = \{f_{m}^{e}\} + \{T_{m}^{e}\}
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
&[M_{\theta\theta}^{e}]\{\ddot{\theta}^{e}\} + [C_{\theta u}^{e}]\{\dot{u}^{e}\} - \left[C_{\theta\phi}^{e}\right] \\
&\quad \times\left(\left[K_{\phi\phi}^{e}\right]^{-1}\left(\left[K_{\phi u}^{e}\right]\{\ddot{u}^{e}\} + \left[C_{\phi\theta}^{e}\right]\{\ddot{\theta}^{e}\} + \left[K_{\phi\theta}^{e}\right]\{\dot{\theta}^{e}\}\right)\right) \\
&\quad + [C_{\theta\theta}^{e}]\{\dot{\theta}^{e}\} + [K_{\theta\theta}^{e}]\{\theta^{e}\} = -\{T_{\theta}^{e}\}
\end{aligned}
\tag{24}
$$

The equations (23) and (24) can be written in the matrix form as follows:

$$
\begin{aligned}
&\begin{bmatrix} M_{uu}^{e} & 0 \\ 0 & M_{\theta\theta}^{*} \end{bmatrix}\begin{Bmatrix} \ddot{u}^{e} \\ \ddot{\theta}^{e} \end{Bmatrix} + \begin{bmatrix} C_{uu}^{*} & C_{u\theta}^{*} \\ C_{\theta u}^{*} & C_{\theta\theta}^{*} \end{bmatrix}\begin{Bmatrix} \dot{u}^{e} \\ \dot{\theta}^{e} \end{Bmatrix} + \begin{bmatrix} K_{uu}^{*} & K_{u\theta}^{*} \\ 0 & K_{\theta\theta}^{*} \end{bmatrix}\begin{Bmatrix} u^{e} \\ \theta^{e} \end{Bmatrix} \\
&= \begin{Bmatrix} \{f_{m}^{e}\} + \{T_{m}^{e}\} \\ -\{T_{\theta}^{e}\} \end{Bmatrix}
\end{aligned}
\tag{25}
$$

where:

$$
\begin{aligned}
\left[M_{\theta\theta}^{*}\right] &= [M_{\theta\theta}^{e}] - \left[C_{\theta\phi}^{e}\right]\left[K_{\phi\phi}^{e}\right]^{-1}\left[C_{\phi\theta}^{e}\right] \\
\left[C_{\theta\theta}^{*}\right] &= [C_{\theta\theta}^{e}] - \left[C_{\theta\phi}^{e}\right]\left[K_{\phi\phi}^{e}\right]^{-1}\left[K_{\phi\theta}^{e}\right] \\
\left[C_{\theta u}^{*}\right] &= [C_{\theta u}^{e}] - \left[C_{\theta\phi}^{e}\right]\left[K_{\phi\phi}^{e}\right]^{-1}\left[K_{\phi u}^{e}\right] \\
\left[C_{u\theta}^{*}\right] &= -[C_{u\theta}^{e}] + \left[K_{u\phi}^{e}\right]\left[K_{\phi\phi}^{e}\right]^{-1}\left[C_{\phi\theta}^{e}\right] \\
\left[K_{u\theta}^{*}\right] &= -[K_{u\theta}^{e}] + \left[K_{u\phi}^{e}\right]\left[K_{\phi\phi}^{e}\right]^{-1}\left[K_{\phi\theta}^{e}\right] \\
\left[K_{uu}^{*}\right] &= [K_{uu}^{e}] + \left[K_{u\phi}^{e}\right]\left[K_{\phi\phi}^{e}\right]^{-1}\left[K_{\phi u}^{e}\right] \\
\left[C_{uu}^{*}\right] &= \alpha[K_{uu}^{e}] + \beta[M_{uu}^{e}] \quad \text{(Rayleigh damping)}
\end{aligned}
\tag{26}
$$

The equation (25) written in the simplified form as:

$$
[M_{G}]\{\ddot{\delta}_{g}\} + [C_{G}]\{\dot{\delta}_{g}\} + [K_{G}]\{\delta_{g}\} = \{F_{G}\}
\tag{27}
$$

2.2 Thermo-visco-elastic formulation

In most of the problems, elastic behaviour is assumed, but memory effects, both in the mechanical and thermal response, i.e. viscoelasticity and second sound, are also discussed (Bargmann, 1974). This reflects the intense interest, which has been shown earlier days in this field, owing to the great practical importance of dynamic effects in modern aeronautics and astronautics, nuclear reactors, high-energy particle accelerators, and its potential importance in cryogenic applications. A finite element formulation accounting both the temperature and memory (time) effects for axisymmetric and plain strain problems were presented by Srinatha and Lewis (1981). Plane stress finite element solution of beam without considering temperature effect was given by Johnson and Tessler (1995). In the present formulation viscoelastic plane stress finite element is developed based on the constitutive law given by Johnson and Tessler (1995) and thermal effects are incorporated in the model by following the procedure given by Srinatha and Lewis (1981). Viscoelastic materials have time dependent stiffness coefficients $[C^{*}(t)]$:

$$[C^{*}(t)] = [C]G_0(t) \tag{28}$$

At $t = 0$, $[C^{*}(0)] = [C]$. $G_0\ (t)$ is relaxation law and is specific for a given material, suffix "0" indicates that $G_0\ (t)$ is valid at stress free reference temperature. The relaxation modulus can be expressed in terms of "Prony series" (Zienkiewicz and Taylor, 2000) as:

$$G_0(t) = \mu_0 + \sum_{i=1}^{n} \mu_n e^{-t/t_n} \tag{29}$$

t_n are called viscoelastic relaxation times and $\sum_{i=1}^{n} \mu_n = 1$ where n is the number of terms chosen in Prony series to represent the relaxation law. In general, $n = 2, 3$ may give a reasonable approximation. The constitutive relation for a linear viscoelastic solid under plane stress following the steps of Johnson and Tessler (1995) and Christensen (1982) can be written as:

$$\{\sigma\} = [C]\{\varepsilon\} + [C]\int_{\tau=-\infty}^{t} \frac{\partial G_0(t-\tau)}{\partial \tau}\{\varepsilon\}\partial\tau \tag{30}$$

The first term on RHS of the above equation represents instantaneous elastic response and second term represents history dependent part. To bring the temperature effects into the model, the relaxation modulus has to be a function of temperature. For this purpose, the material is assumed to be "thermorheologically simple." It means that a uniform change in temperature of the body leads to a corresponding shift in relaxation function on logarithmic time scale:

$$G_0(t) \equiv L(\log(t)) \tag{31}$$

In the above equation t is time and L is Laplace transform, thermorheological simplicity means that can be expressed as follows:

$$G_T(t) = L[\log(t) + A(T)] \tag{32}$$

$G_T(t)$ is relaxation law valid at temperature T and $A(T)$ is called temperature shift function. The shift function obeys the following properties.

Setting:

$$A(0) = 0 \text{ and } \frac{\partial A}{\partial T} > 0 \tag{33}$$

$$A(T) = \log(\psi(T)) \text{ where } \psi \text{ also obeys } \psi(0) = 0 \text{ and } \frac{\partial \psi}{\partial T} > 0 \tag{34}$$

equation (32) can be written as:

$$G_T(t) = L[\log(t) + \log(\psi(T))] = L[\log(\xi)] = G_0(\xi) \tag{35}$$

ξ is called "reduced time" given as follows:

$$\xi = t\psi(T) \tag{36}$$

Equation (35) states that relaxation modulus at any given temperature can be obtained from the modulus at base temperature by simply replacing the time t by reduced time ξ. This is only possible by postulating that material is thermorheologically simple. The above formulae are valid when the body is in isothermal condition, above the stress free temperature. When the body undergoes an arbitrary transient temperature change, the reduced time is defined as:

$$\xi = f(x, t) = \int_0^t \psi[T(x, \lambda)]d\lambda \tag{37}$$

x is any special point. In finite element ξ is calculated for each element considering the element's average temperature. Hence, the constitutive law for a viscoelastic material undergoing arbitrary transient temperature change can be written as:

$$\begin{aligned}\{\sigma\} &= [C]\{\varepsilon\} - \{a\}\{\theta\} \\ &+ \int_{\tau=-\infty}^{t} \left\{ [C]\frac{\partial G_0(\xi - \xi_\tau)}{\partial \tau}\{\varepsilon\} - \{a\}\frac{\partial G_0(\xi - \xi_\tau)}{\partial \tau}\{\theta\} \right\} d\tau\end{aligned} \tag{38}$$

where $\xi = f(x, t)$, the effect of integral term appearing in the above equation can be turned into what is known as "memory load" while handling in finite element:

$$\begin{aligned}\{F_M\} &= \{F_{M1}\} - \{F_{M2}\} \\ \{F_{M1}\} &= \int_V [B_1]^T[C][B_1]\int_{\tau=-\infty}^{t} \frac{\partial G_0(\xi - \xi_\tau)}{\partial \tau}\{u_e\}d\tau dV \\ \{F_{M2}\} &= \int_V [B_1]^T[a][N_2]\int_{\tau=-\infty}^{t} \frac{\partial G_0(\xi - \xi_\tau)}{\partial \tau}\{\theta_e\}d\tau dV\end{aligned} \tag{39}$$

The time integrals appearing in equation (39) can be evaluated numerically using trapezoidal rule (Srinatha and Lewis, 1981). For the k-th time interval:

$$\int_{\tau=-\infty}^{t} \frac{\partial G_0(\xi-\xi_\tau)}{\partial \tau}\{u_e\}d\tau = \sum_{i=1}^{k-1}[G_0(\xi_k-\xi_{i+1})-G_0(\xi_k-\xi_i)]\{u_e^{*i}\}$$
$$\int_{\tau=-\infty}^{t} \frac{\partial G_0(\xi-\xi_\tau)}{\partial \tau}\{\theta_e\}d\tau = \sum_{i=1}^{k-1}[G_0(\xi_k-\xi_{i+1})-G_0(\xi_k-\xi_i)]\{\theta_e^{*i}\} \tag{40}$$

where:

$$\{u_e^{*i}\} = \frac{1}{2}\{u_e^i + u_e^{i+1}\} \;\; and \;\; \{\theta_e^{*i}\} = \frac{1}{2}\{\theta_e^i + \theta_e^{i+1}\} \tag{41}$$

equation (40) shows that for evaluating memory load at current time step it is required to store the displacement and temperature history up to that time step. A more efficient way of doing this integration is by setting a recursive relation by making use of Prony series (equation (29)). Substituting equation (29) in equation (40) it follows that the RHS of equation (40) gives:

$$\sum_{j=1}^{n}\mu_j\sum_{i=1}^{k-1}e^{-(\xi_k/t_j)}\left[e^{\xi_{i+1}/t_j}-e^{\xi_i/t_j}\right]\{u_e^{*i}\} = \sum_{j=1}^{n}\mu_j\{q_{j,k}\}$$
$$\sum_{j=1}^{n}\mu_j\sum_{i=1}^{k-1}e^{-(\xi_k/t_j)}\left[e^{\xi_{i+1}/t_j}-e^{\xi_i/t_j}\right]\{\theta_e^{*i}\} = \sum_{j=1}^{n}\mu_j\{q'_{j,k}\} \tag{42}$$

with recursive relations for $\{q_{j,k}\}$ and $\{q'_{j,k}\}$ as:

$$\{q_{j,k}\} = e^{-(\xi_k-\xi_{k-1})/t_j}[1-e^{-(\xi_{k-1}-\xi_{k-2})/t_j}]\{u_e^{*k-2}\} + \{q_{j,k-1}\}$$
$$\{q'_{j,k}\} = e^{-(\xi_k-\xi_{k-1})/t_j}[1-e^{-(\xi_{k-1}-\xi_{k-2})/t_j}]\{\theta_e^{*k-2}\} + \{q'_{j,k-1}\} \tag{43}$$

now the time integrals in equation (40) can be easily be evaluated. After evaluating the integrals, the memory loads are obtained from equation (39). The memory load is added to the mechanical load in equation (25).

2.3 Solution procedure

The coupled equation is solved after assembly using Newmark β method (Bathe, 1997) with a time step of 1 ms. The memory load is added to the RHS of the equation (25), hence in equation (25), for n-th time interval the mechanical loading is given by:

$$\{f_m\}_n = \{F_{M1}\}_{n-1} - \{F_{M2}\}_{n-1} \tag{44}$$

3. Results and discussion

The response of cantilever piezo-thermo-viscoelastic composite beam under thermal shock is studied. The effect of memory load and temperature dependent relaxation law on the response of composite beam is considered. Two types of viscoelastic materials (DYAD609 and DYAD606) are used in the present study. The effect of viscoelastic core thickness (t_c) on the response of the system is investigated. Figure 1 shows the physical configuration of a piezo-thermo-viscoelastic composite beam. The base beam is made up of graphite-epoxy composite material and the piezo layer, which is bonded to base beam is used as a sensor. The viscoelastic core is used in between the sensor and actuator. The piezo layer, which is above the viscoelastic layer, is called actuator. Generally, this type of treatment is called as active constrained layer damping (ACLD) treatment. All the layers are firmly bonded to each other. It is assumed that there is no slip between the layers. Thermal boundary conditions are shown in Figure 1. The geometric parameters of beam model and material properties are tabulated in Tables I and II.

It is well-known that the cantilever composite beam undergoes large vibration than other configurations, so the cantilever beam is considered for the present analysis. The primary objective of the study is to compare the system response with and without viscoelastic relaxation under thermal environment by considering the thermal relaxation times. The response is taken at the tip of the free end of cantilever beam.

The relaxation function of DYAD 609 and DYAD606 viscoelastic core materials at different temperature in frequency domain is given by Nashif *et al.* (1985). The relaxation function in time domain is obtained by transforming the data in frequency domain to the time domain by following the procedure of Christensen (1982). The relaxation function in time domain for DYAD 609 and DYAD606 viscoelastic material at reference temperature 24 and 10°C, respectively, are given as follows:

$$G_o(t) = 0.008 + 0.783e^{-t/0.00532} + 0.21e^{-t/0.05002} \quad \text{(DYAD 609)} \tag{45a}$$

$$G_o(t) = 0.006 + 0.21e^{(-t/0.03706)} + 0.784e^{(-t/0.00253)} \quad \text{(DYAD 606)} \tag{45b}$$

The temperature shift factor $\psi(T)$ is obtained by curve fitting, using the relaxation functions defined at 24, 38, 66°C for DYAD 609 and 10, 38, 93°C for DYAD 606 viscoelastic materials (Nashif *et al.*, 1985):

$$\psi(T) = -4.06608 + 0.20992T + 4.84694 \times 10^{-5}T^2 \quad \text{(DYAD 609)} \tag{46a}$$

$$\psi(T) = -0.05635 + 0.11313T - 7.49804 \times 10^{-4}T^2 \quad \text{(DYAD 606)} \tag{46b}$$

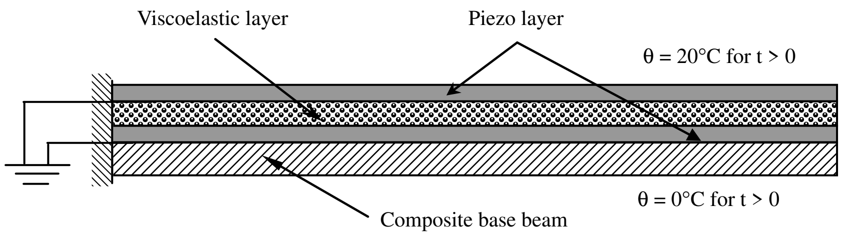


Figure 1. Schematic diagram of piezo-thermo-viscoelastic composite beam

Table I. Geometry of piezo-thermo-viscoelastic composite beam

Length (L)	1 m
Thickness of base beam	$t_b = 6$ mm
Thickness of piezo layer	$t_p = 254\ \mu$m
Thickness of viscoelastic layer	$t_c = 1$ mm

Properties	Graphite/epoxy	Cadmium selenide	Viscoelastic material DYAD 609	DYAD 606
Thermal conductivity (W/m K)				
k_{11}	4.62	2.5	0.18	0.18
k_{22}	0.72			
Specific heat (J/Kg K)	998.12	420	2,000	2,000
Density (kg/m^3)	1,603	7,600	1,250	1,300
Stiffness coefficients (MPa)				
C_{11}	140,200	74,000	1,100	1,352
C_{12}	3,905	45,000	900	579
C_{13}	3,905	39,000	900	579
C_{22}	9,954.9	74,000	1,100	1,352
C_{23}	3,062.5	39,000	900	579
C_{33}	9,954.9	83,000	1,100	1,352
C_{44}	3,447	13,000	100	313
$C_{55} = C_{66}$	7,100	13,000	100	313
Piezoelectric coupling coefficients (C/m^2)				
e_{31}	–	−0.16		–
e_{33}	–	0.347		–
e_{15}	–	−0.138		–
Dielectric constants 10^{-12} (C/V m)				
p_{11}	–	82.6		–
p_{33}	–	90.3		–
Stress temperature coefficients (MPa/K)				
a_{11}	0.162	0.621	1.45	1.25
a_{22}	0.338	0.621	1.45	1.25
Pyroelectric coefficients (μC/K m^2)				
d_{31}	–	−2.94		–
d_{33}	–	−2.94		–
Relaxation times (s)				
τ_1	0	0.01	0	0
τ_2	0	0.05	0	0

Sources: Tiahanu *et al.* (2002); Nashif *et al.* (1985); Chang and Shyong (1994)

Table II. Properties of composite, piezoelectric, and viscoelastic materials

The relaxation function at any other temperature is obtained by using the shift factor in the equation (32). Figures 2 and 3 shows the relaxation function and shift factor of DYAD 609 and DYAD 606 viscoelastic material at different temperature. The postulate of thermo-rheological simplicity can be verified from these curves. The meaning of this is change in temperature causes a uniform shift of relaxation function on a logarithmic time scale. The properties of viscoelastic material are given in the Table II.

In the present analysis Newmark β method is used to solve the dynamic system. Initially, the time step is selected as 0.01, which is selected, based on $1/20f$ (f is lowest natural frequency) and transient response is obtained. To find the influence of smaller time steps, the beam analyzed for 0.1, 0.01, 0.001 and 0.0001 s time steps also. From the Figure 4, it is clear that there is no significant change in the transient response for time step beyond 0.001 s. So, the time step has been taken as 0.001 s for all the analysis carried out in the paper.

The relaxation time represents the time-lag needed to establish steady state heat conduction in an element of volume when a temperature gradient is suddenly imposed

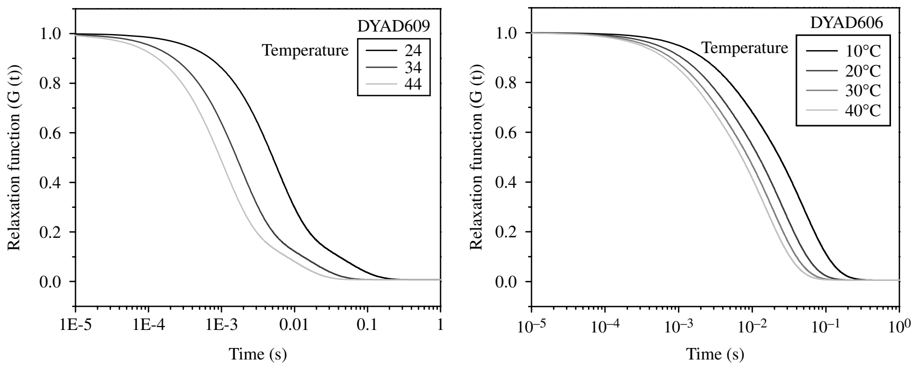


Figure 2.
Relaxation function of a sound coat DYAD609 and DYAD606 viscoelastic material

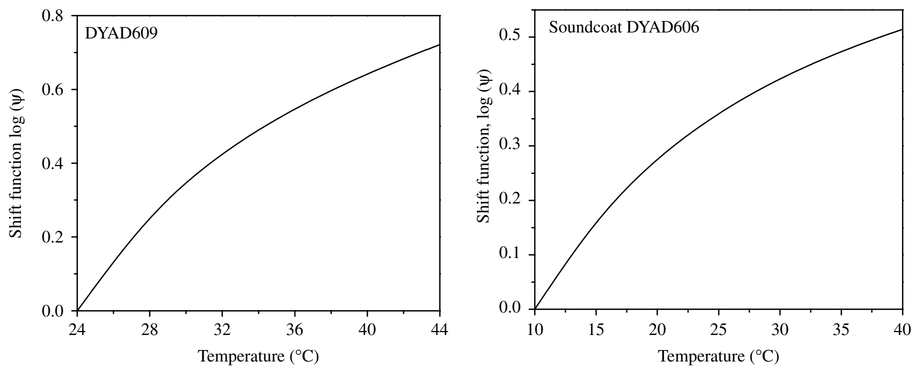


Figure 3.
Shift function of a sound coat DYAD609 and DYAD606 viscoelastic material

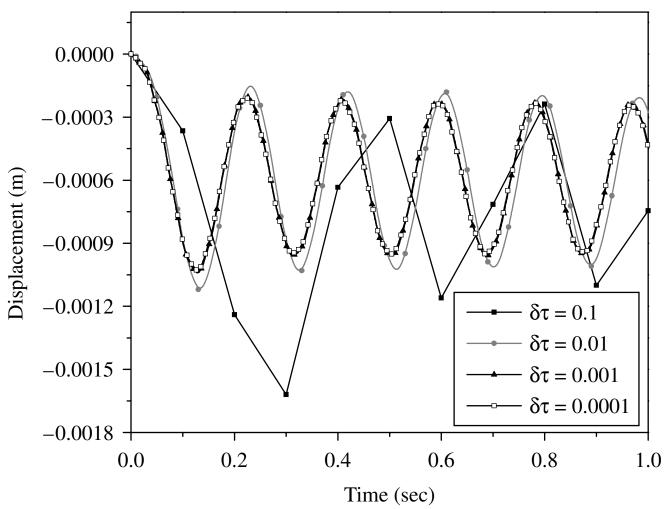


Figure 4.
Transient response of sandwich beam at different time steps

on that element (Lord and Shulman, 1967). The magnitude of the relaxation time has been estimated for particular types of collision processes, the details are given by Peierls (1955). In present case, the relaxation time for piezo layers taken from the reference (Tiahanu *et al.*, 2002), for other layers (base and visco layers) it is taken as zero. In the present work, the relaxation time have been taken to the particular material based on literature. Apparently, the results are converged to 0.001 s, which is 1/10th and 1/50th (0.01 and 0.05 s) of the relaxation times used in the study.

3.1 Validation

In order to ensure the correctness of the present formulation, the results are taken for one typical case of a piezo-thermo-viscoelastic isotropic beam and validated with Pradeep (2006) as shown in Figure 5.

The base beam is made up of mild steel and the properties and geometry of other layers used for validation are given in the Tables I and II. From the Figure 5 it is observed that, results obtained using the present formulation are in good agreement with the results presented by Pradeep (2006). Hence, the present formulation can be used to study the response of piezo-thermo-viscoelastic composite beam.

3.2 Mechanical response of a piezo-thermo-viscoelastic beam

Figure 6 shows the mechanical response of piezo-thermo-viscoelastic cantilever composite beam subjected to transient thermal load. The response is obtained for different viscoelastic core thickness, with and without memory load (viscoelastic effect). The response for instantaneous elastic (without memory) case can be obtained by assuming a constant value = 1 for the viscoelastic relaxation law in place of equations (45a) and (45b). Figure 6 shows the comparison of response with memory (black line) and without memory (gray line). It can be seen that there is decay in the response when the memory load is considered. The instantaneous elastic response (without memory) runs away from the actual response. This shows the need for considering the viscoelastic effect. The reason can be attributed to the very quick relaxation of property of DYAD 609 viscoelastic material.

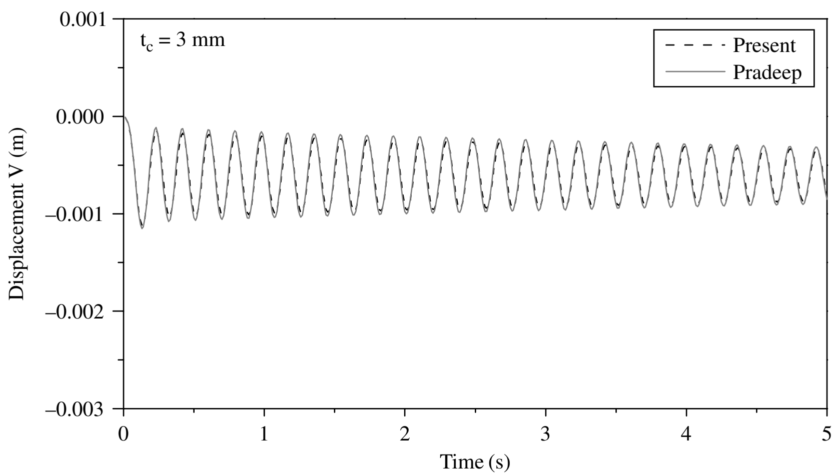


Figure 5.
Validation with isotropic piezo-thermo-viscoelastic clamped free beam with 3 mm core thickness

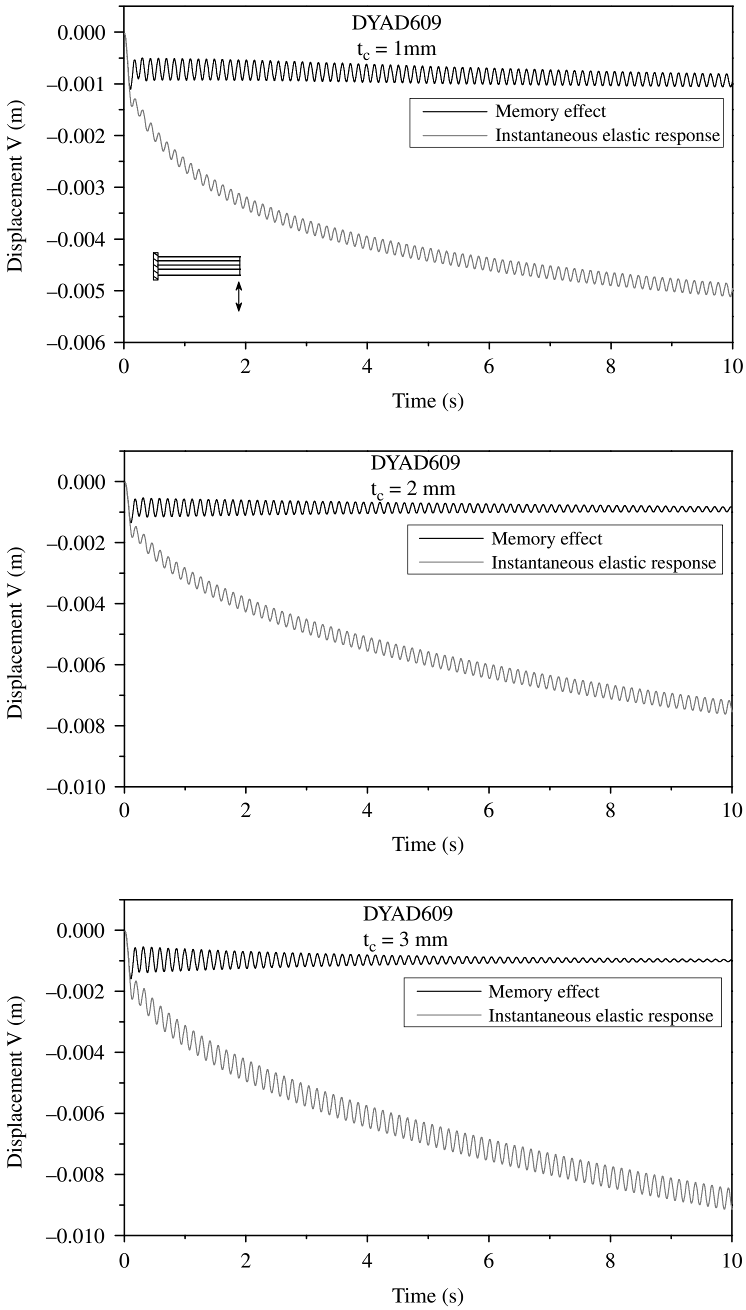


Figure 6.
Effect of viscoelastic memory on mechanical response of a piezo-thermo-viscoelastic composite beam

The viscoelastic relaxation times t_1 and t_2 are the measure of quickness of relaxation. If the relaxation times are small then there is a greater discrepancy between the instantaneous elastic response and viscoelastic response. Also it can be seen from equation (45a) and (45b), if viscoelastic relaxation times are high then the exponential time decaying terms do not contribute much to the relaxation function. On the other hand when viscoelastic relaxation times are small the contribution of exponential terms become high, due to that the response characteristics get drastically affected. In the present case $t_1 = 0.00532$ s and $t_2 = 0.05002$ s (equation (45a)) which are very small compared to response period of interest (10 s). Hence, there is a discrepancy by ignoring the viscoelastic relaxation effects.

If the viscoelastic core thickness increases then damping increases, hence there is more deviation in the instantaneous elastic response. This is because, the more is the viscoelastic content in the system the greater is the importance of memory effect. However, if the response period of interest and viscoelastic relaxation times is closer to each other then there is no deviation between instantaneous elastic response and viscoelastic response (with and without memory) as shown in Figure 7 with a magnified view.

In Figure 7, the response is plotted for a period of 0.2 s, it is clearly noticed that there is no deviation in the instantaneous elastic and viscoelastic responses up to second viscoelastic relaxation time ($t_2 = 0.05$ s).

3.3 Electric response of piezo-thermo-viscoelastic cantilever beam at different locations across the clamped end of beam

Figure 8 shows the variation of electric potential at different locations of a cantilever piezo-thermo-viscoelastic composite beam. The results are taken for three different thickness of the core when the beam is subjected to transient thermal environment. The potential generated in the top piezoelectric layer (Figure 8(a)) is mostly due to pyroelectric effect and hence will be mostly dictated by upper surface temperature effects. In the sensor layer (Figure 8(b)) with increase in core thickness heat takes time

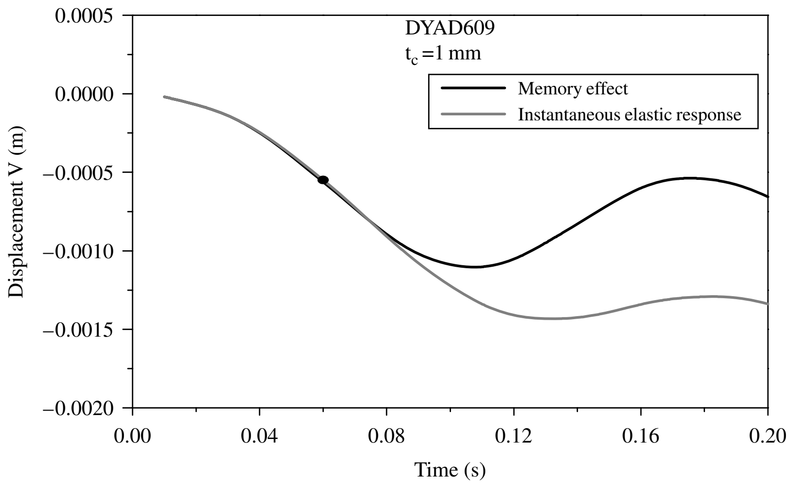


Figure 7.
Effect of viscoelastic memory on mechanical response of a piezo-thermo-viscoelastic composite beam in zoomed view

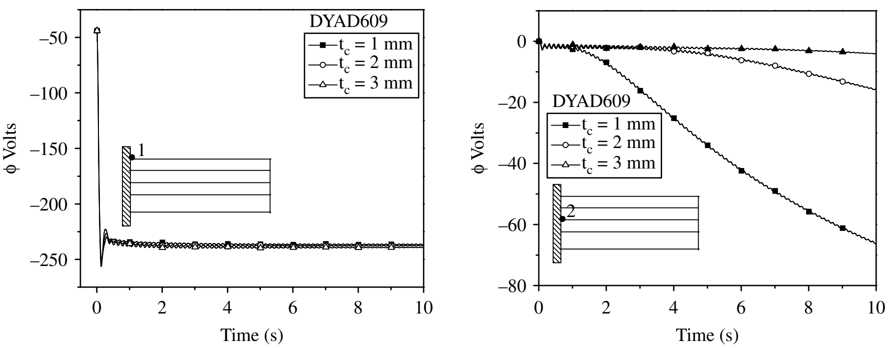


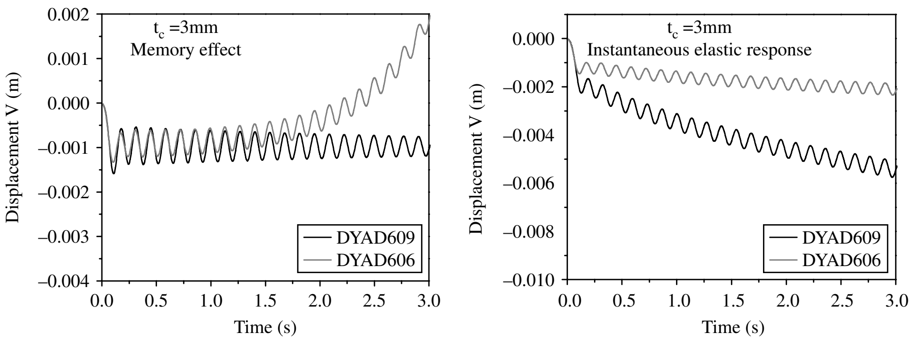
Figure 8. Electric response of a clamped-free piezo-thermo-viscoelastic composite beam

to propagate to the sensor layer and also some amount of the heat dissipation takes place in viscoelastic layer hence only piezoelectric effect is dominant.

3.4 Effect of viscoelastic material on the mechanical response of the system

Figure 9 shows the influence of viscoelastic core material on the mechanical response of piezo-thermo-viscoelastic composite beam. Using DYAD609 and DYAD606 as core materials, the comparison on the mechanical response of cantilever composite beam is made. The thickness of the core material considered is 3 mm. The response is plotted for both, by considering with and without memory effect.

Figure 9(a) shows that when the memory effect is considered the DYAD609 viscoelastic material damping increases with time, and hence the amplitude of vibration decreases. In case of DYAD606 material the response deviates in positive direction because there is a different relaxation times (equation (45a) and (45b)) for DYAD606 and DYAD609 material, which may have effect on the response of the system. Figure 9(b) shows the comparison of response without memory effect, the elastic properties of DYAD606 material is having much influence on damping because of that the response curve deviates from DYAD609 core material.



Notes: (a) Memory effect; (b) instantaneous elastic response

Figure 9. Mechanical response of a DYAD609 and DYAD606 viscoelastic material

4. Conclusions

The transient response of piezo-thermo-viscoelastic composite beam under thermal environment is analyzed. The configuration of the beam is ACLD. The viscoelastic layer of the system is appropriately modeled using "thermorheologically simple material model." The influence of viscoelastic effect "memory effect" on the system response is investigated. There are two types of viscoelastic core materials (DYAD609 and DYAD606) used in the study; the influence of core material and core thickness on the response of the system is also discussed. Some salient features of the present study are as follows:

- The effect of memory load on the system is considerable. Neglecting the memory effect the response of the system deviates from the actual solution. If the response period of interest and viscoelastic relaxation times is same then the influence of memory load may not have effect on the response of the system. In general, the viscoelastic relaxation times are very less than the response period of interest, then the memory effect is very important factor to be considered.
- As the thickness of viscoelastic layer increases the effects of viscoelastic relaxation becomes important in mechanical and electrical response of the system.
- Influence of viscoelastic material is also having more effect on the response of the system.

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