Parametric Identification of Structures with Nonlinearities Using Global and Substructure Approaches in the Time Domain

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Abstract: This paper presents further research on the parametric identification of structures with non-linearities in stiffness and damping properties. Parametric identification is carried out using acceleration responses in the time domain and is useful for structural health monitoring. Cubic nonlinearities in springs and quadratic nonlinearities in dampers are considered. Structural parametric identification is modeled as an inverse problem, based on minimizing the difference between measured responses and calculated responses from a mathematical model. The results of both global and substructural identification approaches are compared. The substructural approach allows us to identify a smaller domain while ignoring external parameters, resulting in a reduced model, but on the other hand the formulation is more complex. Genetic algorithms (GA) are used for filtering the unknown parameter values from within a given range. Simple real coded GA as well as a superior hybrid version obtained by combining with the Levenberg-Marquardt (LM) have been studied. Several numerical examples, including variations of a 10 DOF non-linear lumped mass system and a 12 member truss with several non-linear tuned mass dampers have been studied. The effect of measurement noise have been considered. The substructural method is shown to be superior overall in terms of speed, accuracy and economy (number of sensors) although the global identification approach implemented in conjunction with hybrid GA performs well in some cases.

Key words: structural health monitoring, parametric identification, substructural identification, nonlinear systems, genetic algorithm, tuned mass dampers.

1. INTRODUCTION

Parametric identification methods can be applied to structures to determine unknown parameters such as mass, stiffness and damping properties based on the numerical analysis (non-destructive) of known inputs and the resulting output measurements of the system. Such identification can be useful for model updating, structural health monitoring and damage assessment. The system is excited with a known force and the output is measured and both input and output signals are used to identify the parameters of the system. Structural identification methods for linear systems have been extensively studied and can be classified under various categories, e.g., frequency, time and modal domains, parametric and nonparametric, classical and non-classical methods (Ghanem and Shinozuka 1995; Koh and See 1994). The earlier classical methods of identification such as the maximum likelihood method, the instrumental variable method and the extended Kalman Filter method require a substantial mathematical approach and are

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gradient based. The disadvantages of these methods have lead to the application of evolutionary algorithms based on heuristic principles, such as Genetic Algorithms (Chakraborty *et al.* 2002; Hao and Xia 2002). In the literature on structural parameter identification, there are few significant mathematical studies to show the uniqueness of typical inverse solutions for large systems (Udwadia 1985; Udvadia and Sharma 1978; Gladewell 1986). However, when dealing with larger sizes, the uniqueness and accuracy of the identification are usually illustrated through extensive numerical studies. If a sufficiently large number of measurements are taken either at time steps or at several spatial locations (or a combination of both), then accurate identification is possible.

Worden and Tomlinson (2001), describe different methods for detection, identification and modelling of non-linear structural dynamic systems. Whereas the identification of linear dynamic systems has progressed considerably using several global and substructural approaches, non-linear dynamic system theory is far less established with corresponding difficulties in identification. The modal behaviour of a non-linear system is significantly different from that of a linear system. For example, non-linear systems can have more than one equilibrium point and the frequency of oscillations dependent on the amplitudes of motion. Also unlike linear systems which exhibit resonance only when the frequency of excitation equals the natural frequency, non-linear systems can resonate when the excitation is reasonably near the resonant frequency. Several methods of non-linear identification such as linearization, time domain, frequency domain and black box modelling are discussed in Kerschen *et al.* (2006). Rice (1995) presented an approach where the underlying non-linear differential equation governing the system may be identified by using a linearization technique. Pilipchuk and Tan (2005) introduced a direct method of system identification and parameter monitoring for a general class of non-linear systems in the time domain based on the operator Lie representations and the corresponding Lie series solutions.

A frequency domain technique, the conditioned reverse path method (Kerschen *et al.* 2003), has been used to identify a continuous non-linear system consisting of an experimental cantilever beam with a geometrical non-linearity. Nayfeh (1985) proposed a parametric identification technique that exploits non-linear resonances and comparisons of the behaviour of the system to be identified with those of known systems. A recent approach in this area is black box modelling. In non-linear black-box modelling, artificial neural networks have received the most attention in non-linear structural dynamics (Saadat *et al.* 2004). The application of evolutionary algorithms such as binary coded GA to non-linear identification has also been carried out (Kristinsson and Dumont 1992; Jiang and Wang 2000). Chang (2006) obtained better estimates of non-linear parameters using an improved multi-crossover GA which used real numbers rather than binary representation.

The Levenberg-Marquardt (LM) method is particularly suited to the optimization of non-linear problems. It is an algorithm combining aspects of both the *Steepest-descent* and *Gauss-Newton* methods. Hanagud *et al.* (1985) identified parameters of single DOF non-linear dynamic system with a stiffness cubic non-linearity using the LM method. Rakesh and Park (1997) used an LM iterative direct method to identify parameters for non-linear system with combined quadratic and cubic stiffness non-linearities. The combination of LM and GA offers the advantages of locating the global maxima by GA and thereafter, accurate gradient climbing by LM. This approach has been applied to finding the material properties of a viscoplastic model using a finite element model (Qu *et al.* 2005).

The substructural identification technique is used to identify the parameters only in regions of interest which results in reducing significantly the number of unknown parameters to be identified; hence convergence and accuracy of estimation can be improved. For substructures, the effect of the input excitation is expressed in terms of the responses at the interfaces with the main structure, and substructural identification may be carried out without measuring the actual input excitation to the structure. A smaller number of measurement sensors is required in comparison with full structure identification. Substructural identification methods can be applied in the time domain (Koh *et al.* 1991; Yun and Lee 1997; Koh *et al.* 2003) using the interface measurements or in the frequency domain (Koh and Shankar 2003) using receptances, in which case the formulation can be modified to use interface or internal measurements.

For the worth being presented in this paper, both global and substructural identification approaches were used to identify the parameters in order to monitor the structure's health. For the identification of the full set of structural parameters real coded Genetic Algorithm (GA) and GA combined with the Levenberg-Marquardt method (CGALM) were used. The substructural identification was carried out by pure GA. As stated in Koh *et al*. (2003) the following factors are often considered in numerical simulation studies to test an identification strategy.

1) The strategy should not require an unreasonably good initial guess for convergence to occur. In this paper for both GA and CGALM the initially guessed values are not required and only search range is specified.
2) In practice, a dynamic response is normally measured using accelerometers. Error is incurred in obtaining velocity and displacement signals by integration. Hence, in this work, only acceleration and velocity signals are used for identification.
3) While accurate measurements are possible due to advances in sensor technology, some noise is still inevitable affecting identification accuracy. The strategy should thus be tested in the presence of I/O noise. In this paper each case is identified while taking account of noise in the responses.
4) Though the more the measurements, the better the results in general, the strategy should not assume a complete set of measurements since this is difficult to achieve in reality. The substructural identification technique is used in the work being described to decrease the number of sensors required for measuring responses.

Full structure identification (GA and CGALM methods) satisfies the first 3 factors and the substructural identification (pure GA) approach satisfies all 4 factors.

2. IDENTIFICATION OF GLOBAL STRUCTURE USING HYBRID GA

Simple real coded GA was used to identify the parameters of systems with nonlinearities. Genetic Algorithms are exploration algorithms based on the mechanism of natural selection and survival of the fittest. GA combines the explorative ability of large search spaces as well as reasonable guided search (Michalwicz 1994). But GA needs increased population sizes and a large number of iterations to identify larger number of parameters, which results in increased computational effort. The Levenberg-Marquardt method can also be used to identify the parameters of nonlinear systems. The Levenberg-Marquardt method is a gradient based local search method. Initial values have great importance in updating the parameters in inverse problems using the LM method. If the initial guess values are poor, the LM method may converge to local minima or diverge from the optimum parameter values. The GA parameters which are used to supply the initial values for LM play an important role in the convergence of the hybrid method. If the accuracy of the GA supplied initial value is insufficient, the LM may not lead to the global optimum. This is a limitation of this method.

To avoid the different limitations of real coded GA and the LM method, a hybrid method which combines both real coded GA and a gradient based classical method, Levenberg-Marquardt method, is used. In order to obtain a good initial guess for the LM parameters, GA has been coupled with the LM algorithm. GA is superior in finding global maxima of the objective function and provides the crucial initial guess values for the LM method. This paper uses the continuous or real number version of the GA which does not require binary coding and decoding and is reportedly superior to binary GA when dealing with real number problems. The CGALM procedure is clearly explained in Kishore Kumar *et al*. (2007) and its application for the identification of the parameters of nonlinear systems.

Inverse problems are solved by numerically simulating an experiment, with the mathematical model of a known dynamic system as reference. The experimental acceleration measurements are "generated" from this mathematical model of known stiffness and damping parameters. Noise is added to these measurements for realism. Now, the identification is posed as an inverse problem. The parameters of the same mathematical model are assumed unknown. They have to be filtered out from a given search range, in such a way as to minimize the difference between the experimentally measured responses and the values predicted from the mathematical model.

3. SUBSTRUCTURE FORMULATION

Substructural identification is a technique whereby only the parameters in the regions of interest are identified. This technique reduces the number of parameters to be identified, decreasing the computational effort and improving the convergence. The substructural formulation shown and used here is the technique used by Koh *et al*. (2003). It is valid for one dimensional systems where two substructures meet at one interface node. It is briefly explained here with reference to the system in Figure 1(a). The equations of motion for the substructure shown in Figure 1(b), are given by,

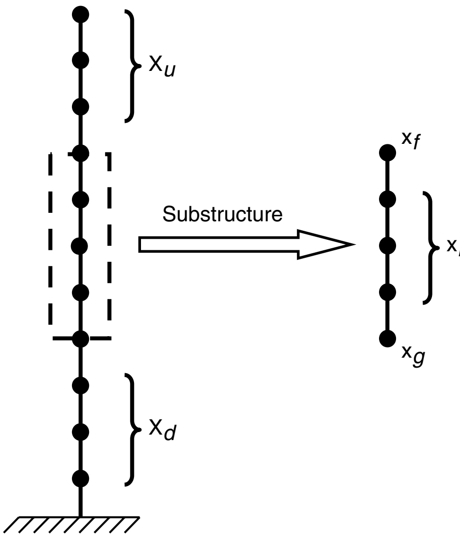

Figure 1. (a) Complete structure; (b) A substructure

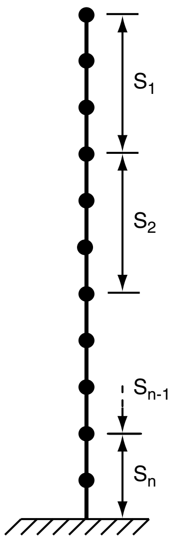

Figure 2. SSI with out overlap

$$\begin{bmatrix} M_{uu} & M_{uf} & & & \\ M_{fu} & M_{ff} & M_{fr} & & \\ & M_{rf} & M_{rr} & M_{rg} & \\ & & M_{gr} & M_{gg} & M_{gd} \\ & & & M_{dg} & M_{dd} \end{bmatrix} \begin{Bmatrix} \ddot{x}_u \\ \ddot{x}_f \\ \ddot{x}_r \\ \ddot{x}_g \\ \ddot{x}_d \end{Bmatrix} + \begin{bmatrix} C_{uu} & C_{uf} & & & \\ C_{fu} & C_{ff} & C_{fr} & & \\ & C_{rf} & C_{rr} & C_{rg} & \\ & & C_{gr} & C_{gg} & C_{gd} \\ & & & C_{dg} & C_{dd} \end{bmatrix} \begin{Bmatrix} \dot{x}_u \\ \dot{x}_f \\ \dot{x}_r \\ \dot{x}_g \\ \dot{x}_d \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{uf} & & & \\ K_{fu} & K_{ff} & K_{fr} & & \\ & K_{rf} & K_{rr} & K_{rg} & \\ & & K_{gr} & K_{gg} & K_{gd} \\ & & & K_{dg} & K_{dd} \end{bmatrix} \begin{Bmatrix} x_u \\ x_f \\ x_r \\ x_g \\ x_d \end{Bmatrix} = \begin{Bmatrix} F_u \\ F_f \\ F_r \\ F_g \\ F_d \end{Bmatrix} \tag{1}$$

Where subscript 'r' denotes internal DOFs for the substructure considered. Subscripts 'f' and 'g' denote interface DOFs of the substructure with the remaining structure on the two sides top and bottom, respectively. Subscripts "u" represents DOFs above the upper interface DOF and subscripts "d" represents DOFs below the lower interface DOF. Let subscript 'j' denote all interface DOFs (i.e. f and g included) for concise presentation. For the substructure considered, the equations of motion may be extracted from the above equations to yield

$$\begin{bmatrix} M_{rj} & M_{rr} \end{bmatrix} \begin{Bmatrix} \ddot{x}_j \\ \ddot{x}_r \end{Bmatrix} + \begin{bmatrix} C_{rj} & C_{rr} \end{bmatrix} \begin{Bmatrix} \dot{x}_j \\ \dot{x}_r \end{Bmatrix} + \begin{bmatrix} K_{rj} & K_{rr} \end{bmatrix} \begin{Bmatrix} x_j \\ x_r \end{Bmatrix} = \{F_r(t)\} \tag{2}$$

Treating interaction effects at the interface ends as "input". The above equation system can be re-arranged as

$$M_{rr}\ddot{x}_r(t) + C_{rr}\dot{x}_r(t) + K_{rr}x_r(t) = F_{\mathrm{r}} - M_{rj}\ddot{x}_j(t) - C_{rj}\dot{x}_j(t) - K_{rj}x_j(t) \tag{3}$$

In the SSI formulation by Koh *et al*. (2003), accelerations, velocities and displacements at the interface DOFs are required as evident in the RHS of Eqn 3. For practical reasons, acceleration is preferred over velocity and displacement. To eliminate the requirement for displacement time signals, the concept of a "quasi-static displacement" vector was adopted. The absolute displacements for internal DOFs were split into "quasi-static" displacements (x_r^s) and "relative" displacements (x_r^*), i.e.

$$x_r(t) = x_r^{\,s}(t) + x_r^{\,*}(t) \tag{4}$$

Quasi-static displacements are obtained by solving Eqn 3 while ignoring the applied force Fr, inertia effect and damping effect (all time-derivative terms set to zero). Hence,

$$K_{rr}x_r^s = -K_{rj}x_j \quad or \quad x_r^s = -K_{rr}^{\;-1}K_{rj}x_j = R\,x_j \tag{5}$$

Here 'R' is called the influence coefficient matrix which relates internal DOFs to interface DOFs under the quasi-static condition. Substituting the above equation into Eqn 3 leads to

$$M_{rr}\ddot{x}_r^*(t) + C_{rr}\dot{x}_r^*(t) + K_{rr}x_r^*(t) = F_r - \left(M_{rj} + M_{rr}r\right)\ddot{x}_j(t) - \left(C_{rj} + C_{rr}r\right)\dot{x}_j(t) \tag{6}$$

The RHS without the Fr term represents forces induced by motion relating to interface DOFs and may be referred to as "interface motion forces" for convenience. In Koh *et al*. (2003), the forces due to the damping were neglected to avoid the measurement of velocity signals at the interface DOFs. But for the identification of nonlinear systems the velocity components can not be neglected as these might affect the identification. Hence both acceleration and velocity signals are measured at the interface DOFs. If there is no excitation within the substructure, Fr simply vanishes and the method can advantageously be used for "output-only" identification (i.e. no force measurement is necessary) for the substructure.

In the substructural identification procedure, the interface acceleration measurements of the substructure have to be obtained from experiment. They are the absolute accelerations $\ddot{\boldsymbol{x}}_j$ (whereby $\dot{\boldsymbol{x}}_j$ is also obtained) and used in the right hand side of Eqn 6. Acceleration measurements are also taken at a few interior points r, for comparison with the values estimated from the model. The relative interior accelerations $\ddot{\boldsymbol{x}}_j^*$ are predicted using Eqn 6 from the model which can be converted to absolute accelerations using Eqn 4.

These estimated accelerations are compared with the measured interior accelerations. The objective function, to be minimized, is a function of the differences in the two types of acceleration.

4. GA PARAMETERS

In the global structure identification scheme, real coded GA is used. Since the number of parameters to be identified in a global model is greater, larger values of population size and the number of generations are necessary, resulting in more computational effort. For global structure identification, the faster hybrid GA (*i.e.* Combined GA and LM method) was also used and compared with the pure GA solution. In CGALM the GA parameters (population size and number of generations) used were small compared to those in the case of pure GA. This is because the purpose of the first stage (real coded GA) of CGALM is just to give acceptable initial guess values to the LM algorithm. The population size and number of generations for CGALM were decided based on the results from Kishore Kumar *et al.* (2007). The GA parameters which are used to supply the initial values for LM play an important role in the convergence of the hybrid method. If the accuracy of the GA supplied initial value is insufficient, the LM may not lead to the global optimum. This is a limitation of this method. Finally the substructural identification technique was used to identify structural parameters in the region of interest around the nonlinearities. Since the number of parameters to be identified here are small, smaller values of GA population were used, thus decreasing the computational effort. In section 6 which present the numerical results, the Tables gives the GA parameters (polulation size, iterations) used in each case.

The identification task can be posed as a minimization problem. For all the approaches a weighted error cost function has been used and is given in the following equation (Koh *et al.* 2003).

$$f = \frac{1}{M}\sum_{i=1}^{M}\mu_i\varepsilon_i \tag{7}$$

Where, $$\varepsilon_i = \frac{1}{L}\sum_{j=1}^{L}\left[\ddot{x}_m(i,j) - \ddot{x}_e(i,j)\right]^2$$

$$\mu_i = \ddot{\chi}/\chi_i, \quad \ddot{\chi} = \sum_{i=1}^{M}\frac{\chi_i}{M} \quad \text{and} \quad \chi_i = \sqrt{\frac{1}{L}\sum_{j=1}^{L}\ddot{x}_m{}^2(i,j)}$$

Here 'M' is the number of measurement sensors used and 'L' is the number of time steps. Subscripts 'm' and 'e' are for measured and estimated responses respectively. Measured responses were simulated numerically using assumed parameter values and noise has been added to compensate the measurement errors. The following is the expression used for adding noise to the responses.

$$\ddot{x}_{noise} = \ddot{x} + \frac{(r-0.5)}{10}\ddot{x} \tag{8}$$

Where 'r' is a random number generated between 0 and 1 with uniform probability and $\ddot{x}_{noise}$ is a signal with noise, $\ddot{x}$ is a signal without noise. A computer with P4 CPU 3.4 GHz and 1.25 GB RAM was used to carry out all the numerical simulations and parametric identification. In the following subsections identification results for each case for both global and substructure approaches are presented. In all cases considered in this paper, in GA algorithm a 40% cross over rate and uniform mutation with a 12% mutation rate were adopted.

5. MODELS STUDIED

In this section, systems and their substructures adopted for identification studies are introduced. Lumped mass systems such as a 10-DOF nonlinear system, a 10-DOF linear system with nonlinear Tuned Mass Dampers (TMDs) and a linear plane truss system with nonlinear TMDs were studied using the methods described earlier, *i. e.*, real coded GA, CGALM and the substructure method.

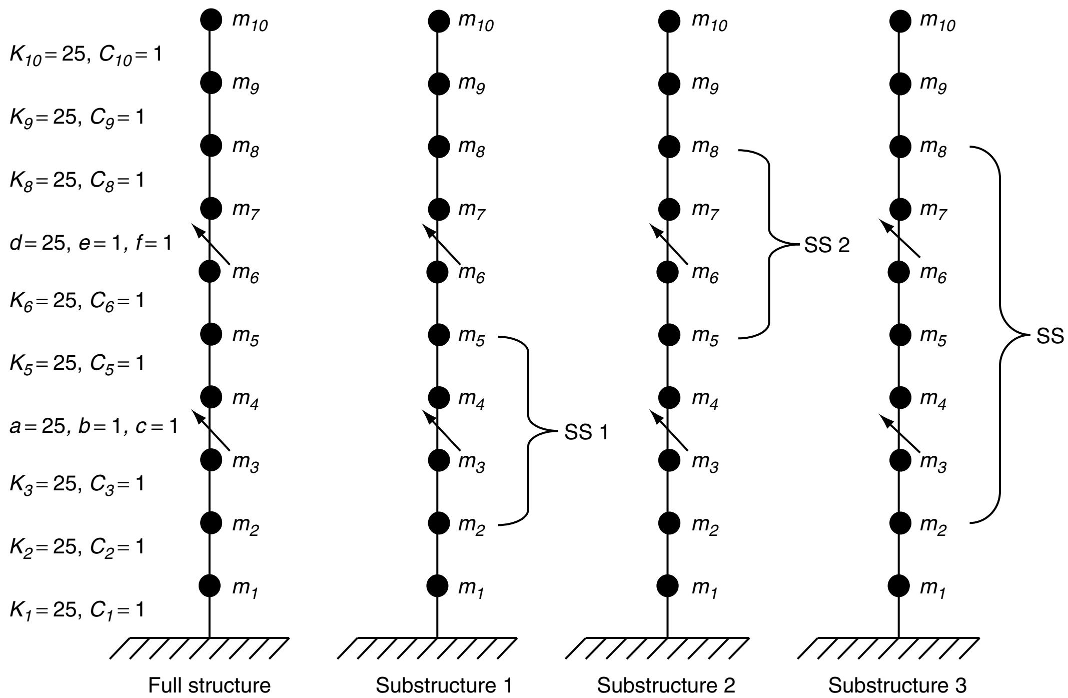

Figure 3. 10-DOF lumped mass system with two nonlinear spring damper pairs

5.1. 10-DOF Nonlinear System

A 10-DOF lumped mass system with two nonlinear spring-damper pairs was considered. All the masses were assumed to be unity *i.e.* 1kg. The spring damper pairs under 4th and 7th DOFs were taken as nonlinear. Figure 3 shows the full system, substructures considered for identification and the actual values assumed to simulate the responses. Cubic nonlinearity (which represents the well known Duffing equation) in spring and quadratic nonlinearity in damping were assumed. The following equation gives the nonlinear relations used.

$$\begin{aligned} k_4(\delta_4) &= a\delta_4 + b\delta_4^{\,3} \\ c_4(\delta_4, \dot{\delta}_4) &= c\dot{\delta}_4(1+\delta_4^{\,2}) \\ k_7(\delta_7) &= d\delta_7 + e\delta_7^{\,3} \\ c_7(\delta_7, \dot{\delta}_7) &= f\dot{\delta}_7(1+\delta_7^{\,2}) \end{aligned} \tag{9}$$

Where 'b' and 'e' are the nonlinear coefficients of the nonlinear spring force expression, 'c' and 'f' are the coefficients of the nonlinear damper force expression, 'δ_4' and 'δ' are the resultant displacements ($\delta_4 = x_4 - x_3$ and $\delta_7 = x_7 - x_6$), '$\dot{\delta}_4$' and '$\dot{\delta}_7$' are the resultant velocities ($\dot{\delta}_4 = \dot{x}_4 - \dot{x}_3$ and $\dot{\delta}_7 = \dot{x}_7 - \dot{x}_6$).

The different identification cases studied here were a) Full structure b) Two substructures, i.e. substructure 1 and substructure 2 considered in such a way that each substructure includes one nonlinear spring-damper pair and c) Substructure 3 which is chosen to include both the non-linear springs and ignore the linear springs. The parameters to be identified were 7 in each of substructure-1 and substructure-2 whereas in substructure 3, the number of parameters to be identified was 14. It can be observed from Figure 3 that for substructure 1, the second and fifth DOFs are the interface and 3rd and 4th DOFs are internal. Similarly internal and interface DOFs for two other substructures can be observed in Figure 3. Eqn 10 gives the dynamic equation for the 10-DOF nonlinear system with two nonlinear spring-damper pairs.

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_7 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_{10} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \\ \ddot{x}_5 \\ \ddot{x}_6 \\ \ddot{x}_7 \\ \ddot{x}_8 \\ \ddot{x}_9 \\ \ddot{x}_{10} \end{Bmatrix}$$

$$
+\begin{bmatrix}
c_1+c_2 & -c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-c_2 & c_2+c_3 & -c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -c_3 & c_3+c(1+\delta_4{}^2) & -c(1+\delta_4{}^2) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -c(1+\delta_4{}^2) & c(1+\delta_4{}^2)+c_5 & -c_5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -c_5 & c_5+c_6 & -c_6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -c_6 & c_6+f(1+\delta_7{}^2) & -f(1+\delta_7{}^2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -f(1+\delta_7{}^2) & f(1+\delta_7{}^2)+c_8 & -c_8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -c_8 & c_8+c_9 & -c_9 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_9 & c_9+c_{10} & -c_{10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_{10} & c_{10}
\end{bmatrix}
\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \end{Bmatrix} \tag{10}
$$

$$
+\begin{bmatrix}
k_1+k_2 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-k_2 & k_2+k_3 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -k_3 & k_3+(a+b\delta_4{}^2) & -(a+b\delta_4{}^2) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -(a+b\delta_4{}^2) & (a+b\delta_4{}^2)+k_5 & -k_5 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -k_5 & k_5+k_6 & -k_6 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -k_6 & k_6+(d+e\delta_7{}^2) & -(d+e\delta_7{}^2) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -(d+e\delta_7{}^2) & (d+e\delta_7{}^2)+k_8 & -k_8 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -k_8 & k_8+k_9 & -k_9 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_9 & k_9+k_{10} & -k_{10} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_{10} & k_{10}
\end{bmatrix}
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{Bmatrix}
=
\begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ F_4(t) \\ F_5(t) \\ F_6(t) \\ F_7(t) \\ F_8(t) \\ F_9(t) \\ F_{10}(t) \end{Bmatrix}
$$

For instance, the governing equations for the substructure 1 shown in Figure 3 can be written as (with reference to Eqn 6)

$$
\begin{bmatrix} m_3 & 0 \\ 0 & m_4 \end{bmatrix}\begin{Bmatrix} \ddot{x}_3{}^* \\ \ddot{x}_4{}^* \end{Bmatrix}
+\begin{bmatrix} c_3+c(1+\delta_4{}^2) & -c(1+\delta_4{}^2) \\ -c(1+\delta_4{}^2) & c(1+\delta_4{}^2)+c_5 \end{bmatrix}\begin{Bmatrix} \dot{x}_3{}^* \\ \dot{x}_4{}^* \end{Bmatrix}
+\begin{bmatrix} k_3+(a+b\delta_4{}^2) & -(a+b\delta_4{}^2) \\ -(a+b\delta_4{}^2) & (a+b\delta_4{}^2)+k_5 \end{bmatrix}\begin{Bmatrix} x_3{}^* \\ x_4{}^* \end{Bmatrix}
$$
$$
=\begin{Bmatrix} F_3(t) \\ F_4(t) \end{Bmatrix}-\left[\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}+\begin{bmatrix} m_3 & 0 \\ 0 & m_4 \end{bmatrix}r\right]\begin{Bmatrix} \ddot{x}_2 \\ \ddot{x}_5 \end{Bmatrix}-\left[\begin{bmatrix} -c_3 & 0 \\ 0 & -c_5 \end{bmatrix}+\begin{bmatrix} c_3+c(1+\delta_4{}^2) & -c(1+\delta_4{}^2) \\ -c(1+\delta_4{}^2) & c(1+\delta_4{}^2)+c_5 \end{bmatrix}r\right]\begin{Bmatrix} \dot{x}_2 \\ \dot{x}_5 \end{Bmatrix} \tag{11}
$$

In substructure 1, the 3rd and 4th DOFs are internal and the 2nd and 5th DOFs are interface. The responses for internal DOFs are obtained using the substructure formulation and the global structure and compared to make sure that the substructure formulation is valid for nonlinear systems. Figure 4 shows the comparison between acceleration responses of the 4th DOF obtained using the global formulation and the substructure formulation (*i.e* the entire structure is divided up into substructures). It can be observed from Figure 4 that both the global and substructure responses match closely and hence the accuracy of the substructure formulation is verified.

The system was excited at the first mass (m_1). The global identification technique was tested for different impulse magnitudes ranging from 5 to 100 N and it identified the parameters exactly. Same analysis was carried out for substructure 1 and the absolute average percentage errors for different impulse magnitudes are given in Table1. From Table 1 one can observe that with an impulse magnitude of 10 N, the substructural approach identifies the parameters more accurately than for other magnitudes of impulse load. Therefore an impulse force of 10 N was applied in the form of initial velocity. The remaining initial conditions were all set to zero.

Table 1. Effect of different magnitudes of impulse load on identification error for substructure 1 in 10-DOF system with two nonlinear spring-damper pairs by GA

Magnitude of impulse load	Avg.% error
5	11.41
10	3.61
30	17.06
50	13.95
100	26.61

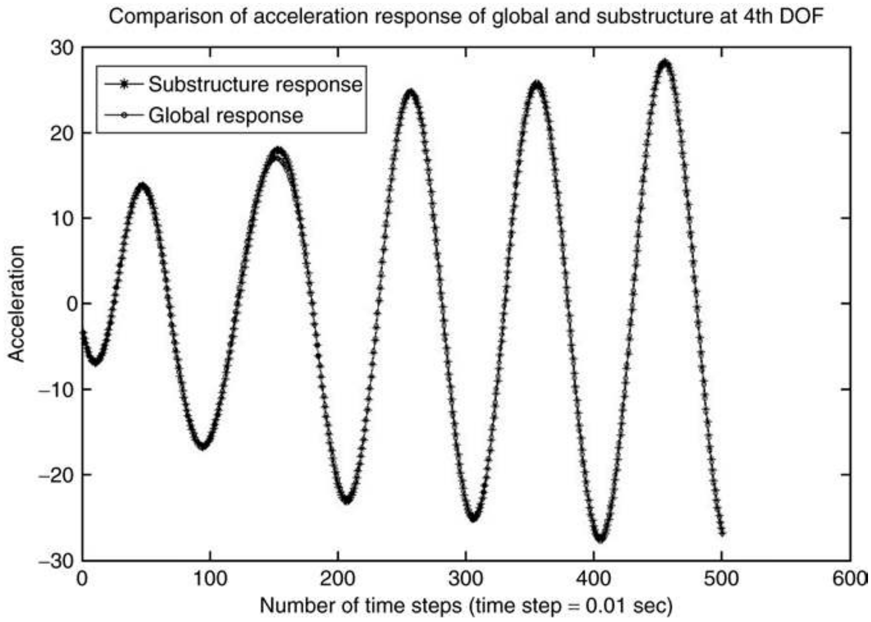

Figure 4. Comparison of acceleration response of global and substructure at 4th DOF of 10-DOF system with two spring-damper pairs

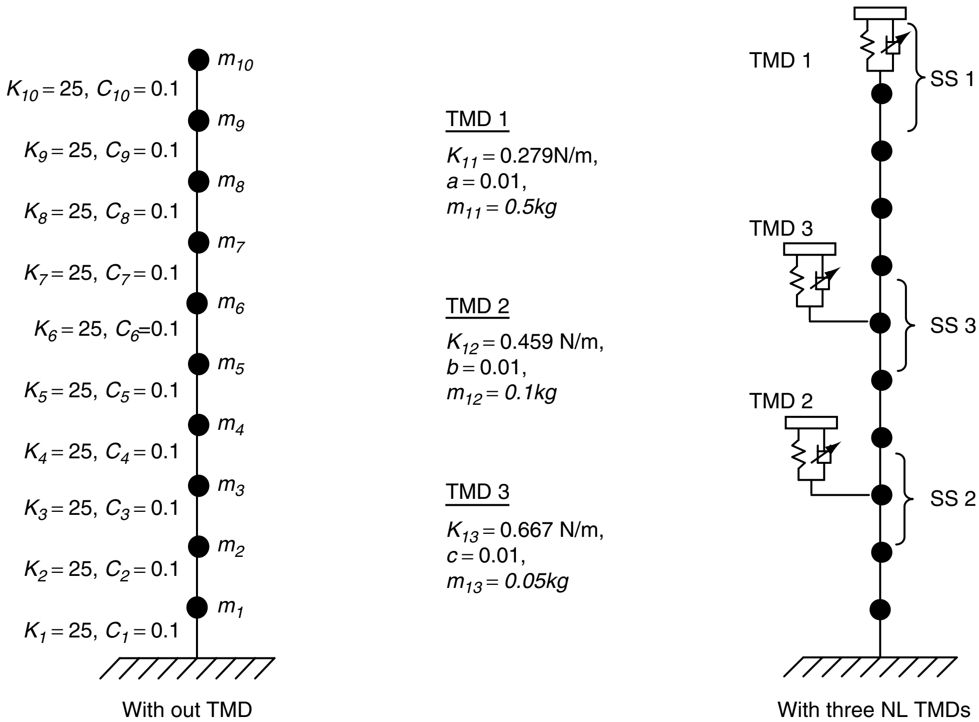

Figure 5. 10-DOF linear system with nonlinear TMDs, substructures considered for identification and assumed parameter values

5.2. 10-DOF Linear System with Nonlinear Tuned Mass Dampers (TMDs)

The system is shown in Figure 5 with a linear primary system and nonlinear TMDs, along with their substructures which were considered for identification. Each substructure includes a non-linear TMD. Tuned mass dampers (TMDs) are secondary elements consisting of a spring, mass and damper added to the primary structure in order to suppress a resonance peak. Usually TMDs are attached to the structures where maximum amplitudes of vibration occur. The three TMDs are each tuned to the first three natural frequencies of the main linear structure. Figure 5 shows the mass and stiffness of each TMD. Here TMDs with nonlinear damper were considered and the substructural identification technique was used for identifying the parameters, *i.e.,*

$$c(\delta, \dot{\delta}) = a\dot{\delta}\,(1+\delta^2) \quad (12)$$

Where δ and $\dot{\delta}$ are resultant displacement and velocity. A 10-DOF linear system with three nonlinear TMDs was considered and the different cases considered are listed below.

(a) 10-DOF linear system with three nonlinear TMDs
(b) Full structure
(c) Substructure 1
(d) Substructure 2
(e) Substructure 3

The system was excited with an impulse load of 10 N at the 8^{th} mass. The stiffness values of the 3 TMDs attached were calculated by tuning them to the first 3 natural frequencies of the main linear structure respectively *viz.,* 0.7473 rad/sec, 2.2252 rad/sec and 3.6534 rad/sec respectively. The dynamic equations for the whole system were obtained by applying D'Alembert's principle. The total mass of all 3 TMDs was 6.5% of the whole mass of the structure, where the first TMD's mass was of 5%, the second TMD's mass was 1% and the third TMD's mass was 0.5% of the whole structure's mass. The nonlinear damping coefficient for every nonlinear damper was assumed as 0.01. All stiffness coefficient values are in N/m and damping coefficient values are in N-s/m. Since the mode shape of a structure gives the relative motion of the DOFs, placement of the TMDs was decided based on the mode shapes of the main structure.

Figure 6 compares the acceleration response between the substructure and global responses of the 3^{rd} DOF of the 10-DOF linear system with three nonlinear TMDs. It can be observed that the response calculated using the global approach exactly matches that of the substructure method, showing the substructure formulation works well for a system with nonlinear TMDs.

5.3. Linear Plane Truss with Nonlinear TMDs

Next, various cases of a linear plane truss systems with one, two and three nonlinear TMDs were considered. The plane truss of 12 members and 6 joints (modelled with 12 linear bar elements) is shown in Figure 7. The truss equations were obtained from the standard stiffness matrix for longitudinally vibrating rods and the TMDs with non-linear dampers were attached to the joints using the force equilibrium conditions at the nodes. The parameters to be identified were the Modulus of Elasticity (E) of each truss element and other TMD parameters. Figures 7 and 8 show different cases analyzed, the substructures considered and the assumed parameter values. The length of both horizontal and vertical bars was taken as 2 meters. Different cases considered were a) a plane truss system without a TMD b) a plane truss system with one nonlinear TMD c) a plane truss system with two nonlinear TMDs d) a plane truss system with three nonlinear TMDs. In each case global and substructure identification was carried out. In Figure 7, for example, the mass of each TMD is 1% of the global structural mass. Stiffness coefficients for each TMD were calculated by tuning to the first 3 natural frequencies of the primary linear structure. Here the first 3 natural frequencies of the linear plane truss were 219.78 rad/sec, 753.39 rad/sec and 933.53 rad/sec respectively. The nonlinear damping coefficient for each TMD is assumed as 4. All the TMDs were constrained to move only in one direction.

Impulse excitation in the form of initial velocity was applied to excite the system as shown in Figure 8, the appropriate value being selected from a range of cases (see Table 2). It can be observed form Table 2 that with an initial velocity (impulse excitation) of 10m/s, parametric identification errors were lower than with other initial velocities.

6. NUMERICAL SIMULATION OF THE ABOVE MODELS – DISCUSSION OF RESULTS

6.1. 10-DOF Nonlinear System

The 10-DOF nonlinear system discussed in section 5.1, with two nonlinear spring-damper pairs and different substructures were considered for identification. The RK 4^{th} order numerical integration method was used to solve the nonlinear dynamic equations.

6.1.1. Global identification approach

Global structure parameters were identified using both simple real coded GA and CGALM. Responses were measured at all the DOFs. Using fewer responses were found significantly affect the accuracy of identification. Table 3 gives the identification results for the global structure and substructure 1 and Table 4 summarizes identification results for both global and substructural

Table 2. Effect of different magnitudes of initial velocities (impulse loads) on identification error for plane truss with one nonlinear TMD by CGALM

Magnitude of initial velocities impulse load)	Avg.% error
5	0.32
10	0.003
20	0.49
25	2.05

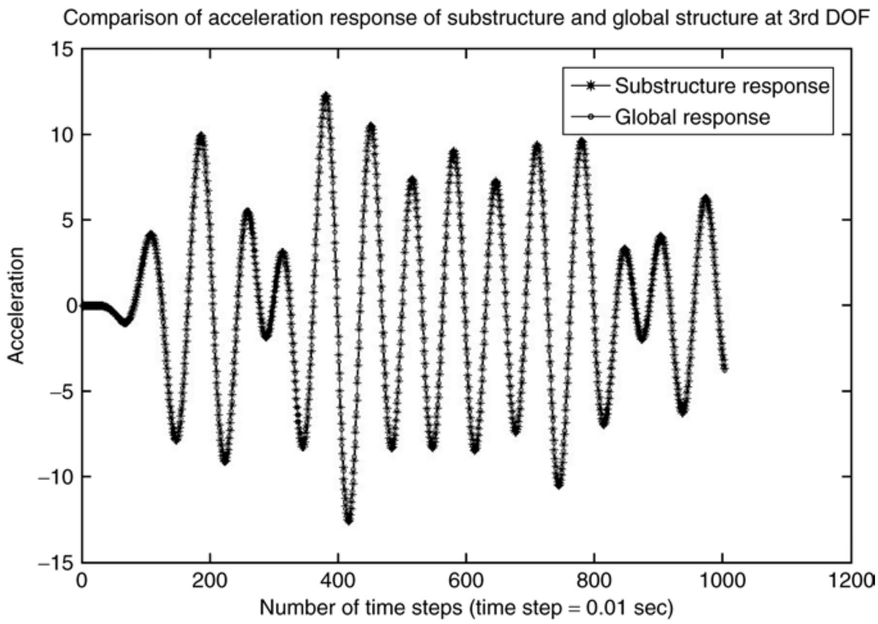

Figure 6. Comparison of acceleration response of global and substructure at 3rd DOF of 10-DOF linear system with three nonlinear TMDs

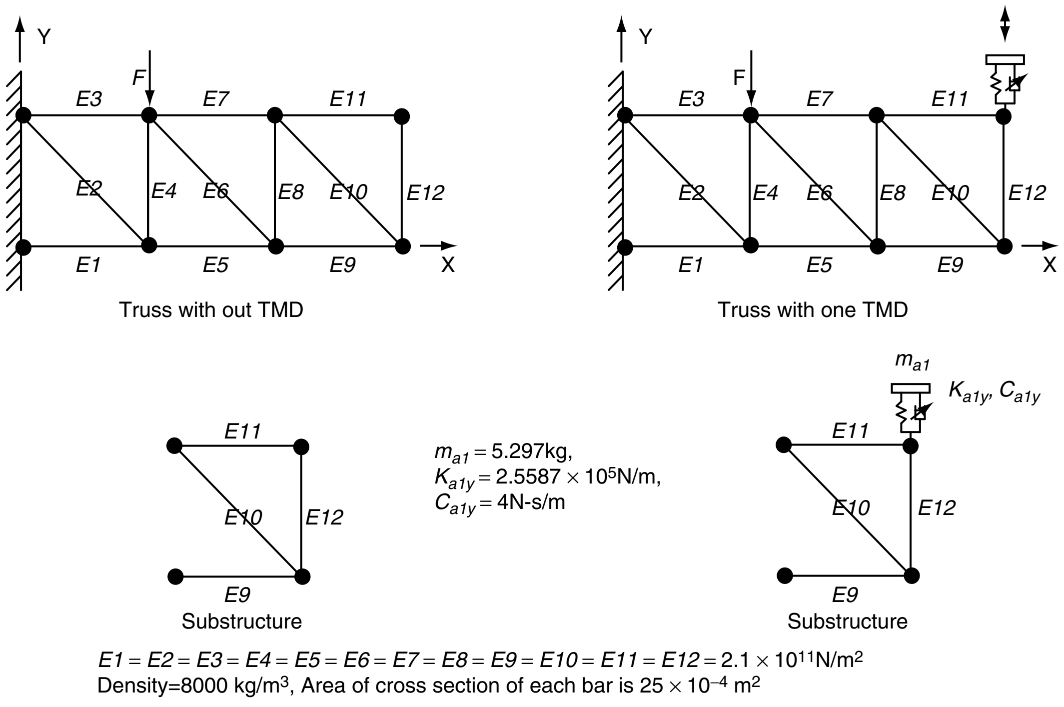

Figure 7. Plane truss system with and without nonlinear TMD, substructures considered and assumed parameter values

Table 3. Identification results for 10-DOF nonlinear system with two nonlinear spring-damper pairs

Parameter	Exact	Estimated (Pure GA)	% error (Pure GA)	Estimated (CGALM)	% error (CGALM)	Estimated (Substructure 1)	% error (Substructure 1)
K_1	25	24.3450	2.62	25.3450	–1.38		
C_1	1	1.1240	–12.40	1.0745	–7.45		
K_2	25	25.2540	–1.02	24.0770	3.69		
C_2	1	0.9924	0.76	0.7469	25.31		
K_3	25	25.4560	–1.82	25.6050	–2.42	25.1290	–0.52
C_3	1	1.0080	–0.80	1.1638	–16.38	1.0536	–5.36
a	25	25.1660	–0.66	24.9340	0.26	26.0490	–4.20
b	1	0.9862	1.38	1.1366	–13.66	0.9913	0.87
c	1	1.0798	–7.98	1.1045	–10.45	1.0631	–6.31
K_5	25	25.3030	–1.21	24.8860	0.46	25.2870	–1.15
C_5	1	0.8862	11.38	0.8135	18.65	1.0955	–9.55
K_6	25	25.7520	–3.01	25.4570	–1.83		
C_6	1	0.8251	17.49	1.2267	–22.67		
d	25	25.0390	–0.16	24.6920	1.23		
e	1	0.9742	2.58	0.9467	5.33		
f	1	1.1533	–15.33	0.8349	16.51		
K_8	25	25.0720	–0.29	25.4290	–1.72		
C_8	1	1.0334	–3.34	1.1495	–14.95		
K_9	25	25.8490	–3.40	24.5710	1.72		
C_9	1	1.0396	–3.96	1.0023	–0.23		
K_{10}	25	25.1350	–0.54	25.0790	–0.32		
C_{10}	1	1.0447	–4.47	0.9025	9.76		

Table 4. Summary of identification results for 10-DOF nonlinear system with two nonlinear spring-damper pairs

	without noise			with ±5% noise		
	Sol. time (sec)	Avg. % error	Max. % error	Avg. % error	Max. % error	GA parameters
Full structure (22 parameters)						
Pure GA	17498	3.2	14.19	4.39	17.49	800, 200
CGALM	773	0	0.002	8.02	25.31	200, 10
Substructure 1 (7 parameters) Time saving compared with full structure **95%**						
Pure GA	780	3.61	–7.81	3.99	–9.55	200, 20
Substructure 2 (7 parameters) Time saving compared with full structure **95%**						
Pure GA	776	3.90	6.28	5.27	8.27	200, 20
Substructure 3 (14 parameters) Time saving compared with full structure **61%**						
Pure GA	6833	2.59	14.21	3.73	8.96	300, 80

identification approaches and GA parameters considered. In CGALM the initial guess values required for the gradient based LM method were supplied using GA with a smaller population size and number of generations.

Since the population size and the number of generations were greater when applying global identification using GA, the solution time longer and the average absolute percentage error was 4.391% with noise *i. e.* the identified results are in well agreement with the assumed parameter values, whereas in CGALM the average absolute percentage error was greater with noise because the second stage (LM method) is sensitive to initial values.

6.1.2. Substructural identification approach

Real coded GA has been used in the substructural identification technique and only responses at the interface and internal DOFs are required. This results in fewer sensors. The identification results are shown in Table 4. The substructural identification approach requires less time. A time saving of 95% was noted compared with that to the global identification approach for substructures 1 and 2. From the point of view of identification accuracy, substructures 1 and 3 proved superior to the global method with simple GA and for all the substructures 1, 2 and 3, the global method with CGALM proved to be better. Figure 9 shows a typical convergence plot for the CGALM method for this case.

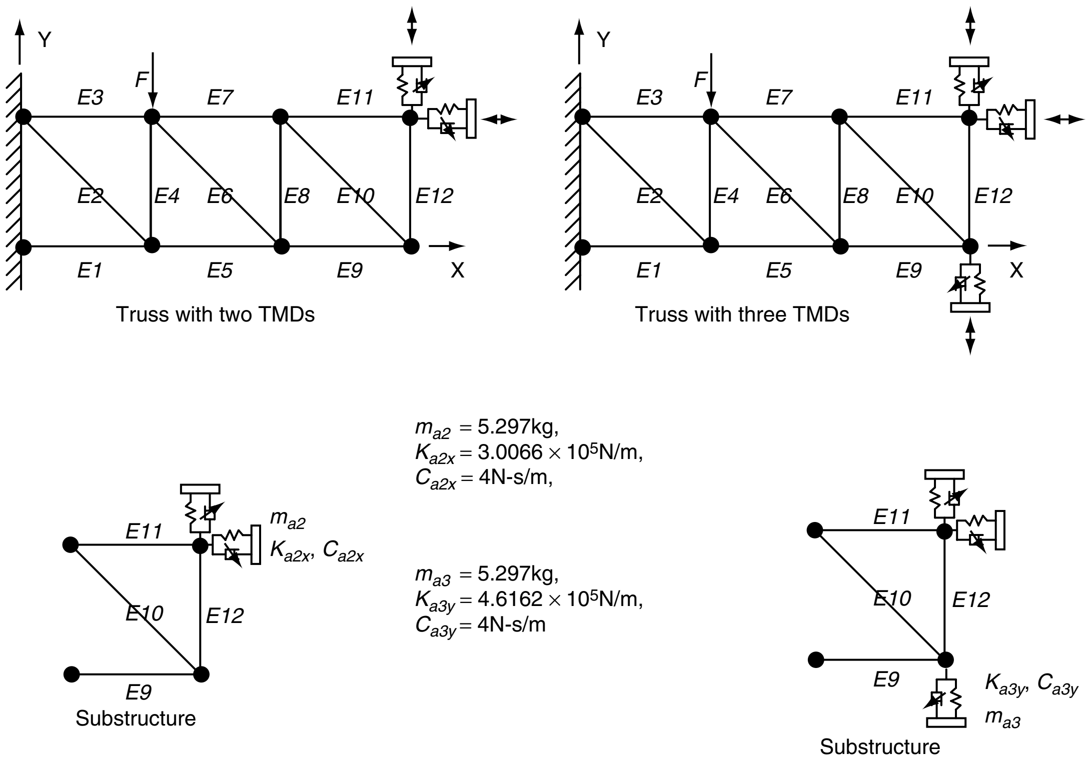

Figure 8. Plane truss system with nonlinear TMD, substructures considered, assumed parameter values

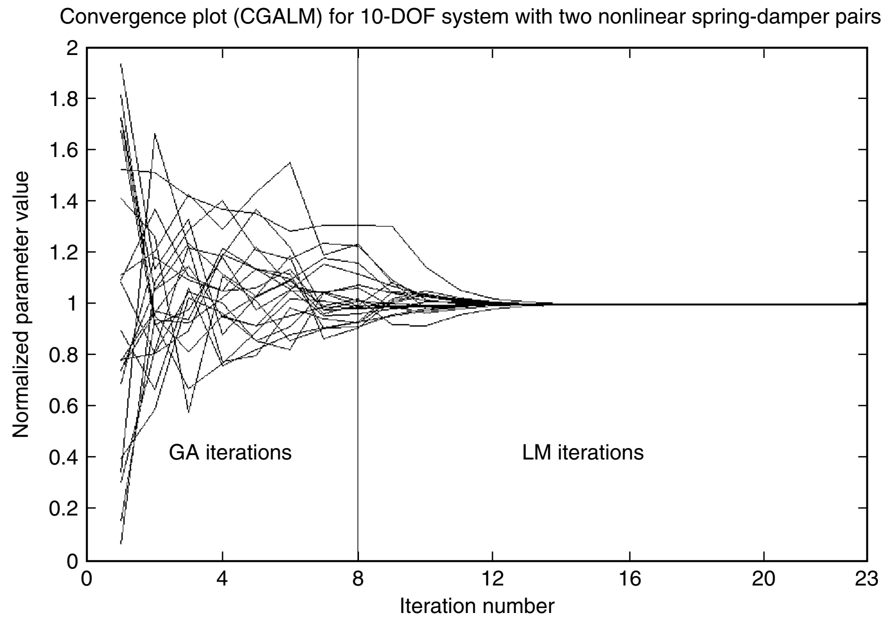

Figure 9. Convergence plot in CGALM method for 10-DOF system with two nonlinear spring-damper pairs

Figure 10 shows the sensitivity analysis for all parameters of the global structure. Each parameter was perturbed from the base values by 1%. The difference between acceleration responses at each DOF for both the perturbed and assumed parameters was calculated and then the RMS value of all the differences at each DOF gives the sensitivity of the parameters. It can be observed that the sensitivities of the nonlinear stiffness coefficients (*b, e*) and all linear and nonlinear damping coefficients are smaller than those of other parameters. Hence it is not easy to identify them accurately.

6.2. A 10-DOF Linear System with Nonlinear Tuned Mass Dampers (TMDs)

The identification of a 10-DOF linear system with nonlinear TMDs, discussed in section 5.2 is presented

Table 5. Summary of identification results of 10-DOF linear system with three nonlinear TMDs at 3rd, 6th and 10th DOFs

	without noise			with ±5% noise		
	Soln. time (Sec)	Avg. % error	Max. % error	Avg. % error	Max. % error	GA parameters
Full Structure (26 parameters)						
CGALM	1602	0.002	0.025	3.83	−24.03	200, 10
Pure GA	15303	5.76	−27.70	5.71	21.09	800, 50
Substructure 1(4 parameters) Time saving compared with full structure **95%**						
Pure GA	773	5.69	−17.09	2.96	−8.94	50, 30
Substructure 2 (6 parameters) Time saving compared with full structure **94%**						
Pure GA	920	3.30	8.24	3.47	−7.54	50, 30
Substructure 3 (6 parameters) Time saving compared with full structure **94%**						
Pure GA	920	4.26	−10.48	6.89	16.02	50, 30

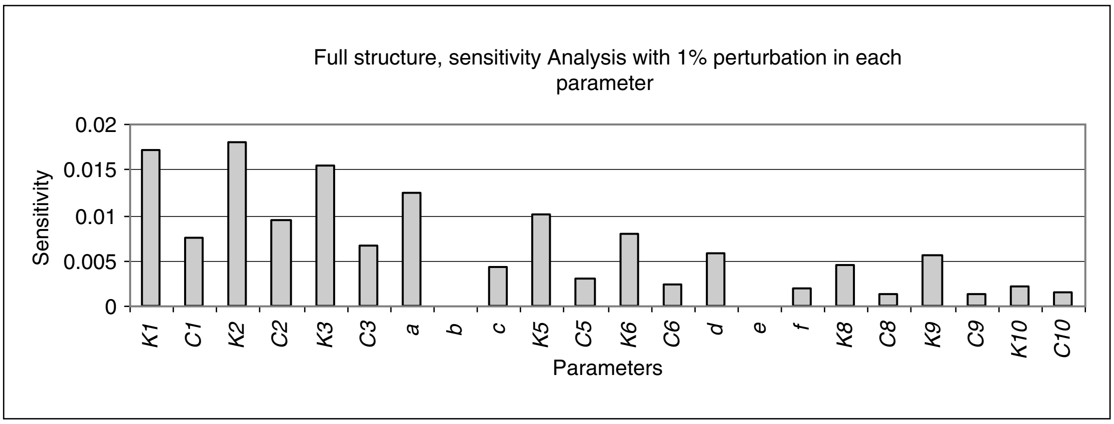

Figure 10. Sensitivity analysis of parameters of full structure, impulse force is applied at first DOF (m1)

here. Three different cases are considered with one, two and three nonlinear TMDs and in each case substructures were chosen which included a TMD. In the following subsections the results of each is discussed clearly.

6.2.1. 10-DOF linear system with three nonlinear TMDs at 3rd, 6th and 10th DOFs

As shown in Figure 5 a 10-DOF linear system has 3 nonlinear TMDs attached which were tuned to the first 3 natural frequencies of the main linear structure. Three substructures were considered for identification purposes in such a way that each substructure included a nonlinear TMD. The identification results summary and the GA parameters (population size and number of generations) considered are shown in Table 5. The substructure approach only requires sensors at the interface and interior, whereas the global approach requires a sensor at every DOF for accuracy.

It can be observed from Table 5 that the identification results for substructures 1 and 2 are better than those for the full structure as obtained by both CGALM and pure GA. About a 95% time saving was obtained for all the substructure cases, in comparison with full structure identification with pure GA. The reason for the greater maximum percentage errors is due to the lesser sensitivity of the substructure parameters.

6.3. Linear Plane Truss with Nonlinear TMDs

The parametric identification of the plane truss with 12 members and 6 joints, discussed in section 5.3, is presented here. For full structure identification using both CGALM and GA, the responses at each joint in both *X* and *Y* directions were measured (a total of 12 responses). In substructure identification by GA only responses at the interfaces and internal joints both in the *X* and *Y* directions were measured which resulted in decrease in the number of sensors required for identification. For all the cases considered the system was excited using an impulse load, which was given in the form of an initial velocity.

6.3.1. Plane truss without TMD

The full-structure and substructures considered for identification and their assumed parameter values are shown in Figure 7. Table 6 gives the identification

Table 6. Summary of identification results of plane truss system with out TMD

	without noise			with ±5% noise		
	Soln. time (Sec)	**Avg. % error**	**Max. % error**	**Avg. % error**	**Max. % error**	**GA parameters**
Full Structure (12 parameters)						
CGALM	469	0.29	−0.81	1.99	−6.06	300, 10
Pure GA	1229	4.18	−11.33	4.70	−19.19	300, 50
Substructure (4 parameters) Time saving compared with full structure **75%**						
Pure GA	301	0.16	−0.47	2.49	3.89	200, 20

Table 7. Summary of identification results of plane truss system with one nonlinear TMD

	without noise			with ±5% noise		
	Soln. time (sec)	**Avg. % error**	**Max. % error**	**Avg. % error**	**Max. % error**	**GA parameters**
Full Structure (14 parameters)						
CGALM	378	0.003	0.014	3.25	15.11	300, 10
Pure GA	4911	2.45	−7.24	3.83	−11.78	400, 80
Substructure (6 parameters) Time saving compared with full structure **78%**						
Pure GA	1054	1.68	−3.39	3.01	10.76	300, 30

results obtained by the both global, substructure approach and the GA parameters assumed. The CGALM identified global parameters with much lower errors with out noise but it needed responses at each DOF (12 sensors). For substructure identification, (without noise) parameters were identified accurately and with only a few measurement sensors (8 sensors). Since the number of parameters to be identified is greater for full structure identification, the percentage of error by GA identification is greater.

6.3.2. Plane truss with one nonlinear TMD

TMD with a nonlinear damper, which was tuned to the first natural frequency of the main linear structure, was attached at the free end of the truss and the TMD was constrained to move in the vertical (y) direction. The first TMD mass and stiffness are m_{a1} and k_{a1y} and are tuned to the first natural frequency of the truss. Figure 7 shows the full structure, substructure and assumed parameter values. The identification results summary for the plane truss with one nonlinear TMD is tabulated in Table 7.

As shown in Table 7, the average percentage error via the substructure approach (with noise) is less than that via the global identification approach, and there is a 78% of time saving as compared with full structure identification by GA. There is also a reduced requirement for the number of sensors for the substructure method, as was the case previously, above.

6.3.3. Plane truss with two nonlinear TMDs

A second TMD, constrained to move in the horizontal (x) direction was added at the free end of the cantilever truss model. The second TMD mass and stiffness are m_{a2} and k_{a2x} and were tuned to the second natural frequency of the truss. Figure 8 shows the full structure, substructure and assumed parameter values. The identification results summary is tabulated in Table 8, giving the time saving and identification errors. The substructure approach performs well compared to that of the CGALM in the presence of noise.

6.3.4. Plane truss with three nonlinear TMDs

In this case a third TMD, constrained to move in the vertical (y) axis was added at the free end. The third TMD mass and stiffness are m_{a3} and k_{a3y} and were tuned to the third natural frequency of the truss. Figure 8 shows the full structure, substructure and assumed parameter values. The identification results summary for the plane truss with one nonlinear TMD is tabulated in Table 9. In this case all 3 approaches with noise (full structure identification by GA, CGALM and substructure approach) identified the parameters with nearly the same degree of accuracy. A time saving of 81% was obtained with the substructure method.

7. CONCLUSIONS

The parametric identification of nonlinear structures in the time domain has been carried out using both global and substructural identification approaches for certain specific numerical examples, ranging from basic lumped mass systems to complex truss type structures. A review of all the cases described in this paper shows a time saving of 61% to 95% when using the substructure method, as opposed to global structure identification with simple GA. This depends on the relative size of the substructure and the number of unknown parameters in the substructure.

Table 8. Summary of identification results of plane truss system with two nonlinear TMDs

	without noise			with ±5% noise		
	Soln. time (sec)	**Avg. % error**	**Max. % error**	**Avg. % error**	**Max. % error**	**GA parameters**
Full Structure (16 parameters)						
CGALM	496	0.041	0.29	7.82	–26.90	300, 10
Pure GA	5985	3.30	–8.61	3.48	–10.05	500, 80
Substructure (8 parameters) Time saving compared with full structure **78%**						
Pure GA	1325	3.02	17.63	6.54	18.99	300, 40

Table 9. Summary of identification results of plane truss system with three nonlinear TMDs

	without noise			with ±5% noise		
	Soln. time (sec)	**Avg. % error**	**Max. % error**	**Avg. % error**	**Max. % error**	**GA parameters**
Full Structure (18 parameters)						
CGALM	707	0.16	1.18	4.26	23.91	300, 10
Pure GA	8183	4.03	–16.08	4.47	12.57	600, 80
Substructure (10 parameters) Time saving compared with full structure **81%**						
Pure GA	1623	4.51	–15.33	4.60	–22.09	300, 50

Generally, the substructure identification approach performs well in the presence of noise and compares well with the global approach in this respect. However in all the cases considered here, the application of the more sophisticated hybrid-GA, *i.e.* combined with the LM algorithm, was able to significantly improve the computational performance of the global identification approach. In this regard it is noted that the presence of noise can cause significant errors with the CGALM, because the second stage of the hybrid algorithm (LM method) is gradient based and is sensitive to the initial values supplied by the GA. From the practical point of view, substructural identification could be preferred because a) fewer of sensors are required when compared with global identification techniques and b) the ability to completely ignore structural parameter values outside the substructure. However, the substructure equations are rather complex to formulate and acceleration responses at the substructure interfaces are required.

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