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Instanton Corrected Non-Supersymmetric Attractors

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Abstract

We discuss non-supersymmetric attractors with an instanton correction in Type IIA string theory compactified on a Calabi-Yau three-fold at large volume. For a stable non-supersymmetric black hole, the attractor point must minimize the effective black hole potential. We study the supersymmetric as well as non-supersymmetric attractors for the D0-D4 system with instanton corrections. We show that in simple models, like the STU model, the flat directions of the mass matrix can be lifted by a suitable choice of the instanton parameters.

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1 Introduction

Some time back, in a seminal work [1], Ferrara, Kallosh and Strominger observed a remarkable feature of asymptotically flat, static, spherically symmetric, extremal black holes in $N = 2$ supergravity coupled to n vector multiplets in four dimensions. Analyzing the spinor conditions, these authors found that the black hole solutions behave as attractors. Although, the scalar fields arising from the vector multiplets take arbitrary values at spatial infinity, they run into a fixed point at the horizon of the black hole. The values of the scalar fields at the fixed point are determined by the magnetic charges of the black hole. This result has subsequently been generalized for dyonic black holes by Strominger [2].

It was soon realized that this apparently simple looking result has remarkable consequences in several aspects of supersymmetric black holes. For example, one might naively think that since the black hole solution in supergravity in general involves scalar fields apart from gauge fields, the area of the black hole and hence it's entropy should depend on these fields. In particular, the macroscopic entropy should depend on the boundary condition imposed on the scalar fields at infinity. However, a microscopic counting of black hole entropy indicates that it depends only upon the electric and magnetic charges of the black hole which are quantized objects. Since the scalar field vacuum expectation values are continuous variables this is a seemingly paradox. This paradox is resolved by the attractor mechanism.

Various properties related to the attractor mechanism in extremal black holes have been investigated extensively afterwards [3–6]. It has been shown that the central charge of the supergravity theory is extremised at the attractor point and the macroscopic entropy of the black holes can be derived from the value of the central charge at the extremum. The problem of finding extremal black hole has been mapped on to an effective one-dimensional theory and the attractor is described in terms of the extremum of the potential in the effective theory. Similar results in $N = 4$ and $N = 8$ supergravity have also been established in some of the above references. Some of the early results on attractor equations in the presence of higher derivative terms are discussed in [7–10]. These results played a key role in arriving at the OSV conjecture on the connection between the macroscopic entropy of black holes and the topological partition function [11].

Multi-centered configurations have been studied and the connection between BPS states in string theory has also been discussed [12, 13].

Another area where important development has taken place in recent times is in the study of the attractor mechanism for non-supersymmetric extremal black holes. It has been realized that the attractor mechanism is actually a consequence of extremality of the black hole and not supersymmetry! Although the existence of a critical point in the moduli space, which need not preserve supersymmetry, had already been pointed out long back [5, 6], not much attention was paid to this class of configurations. A thorough investigation of the attractor mechanism in non-supersymmetric black holes was done only as late as 2005 by Goldstein *et.al.*, [14]. The existence of non-supersymmetric attractors in string theory was first shown in [15]. Subsequently, the attractor equations were written down more elegantly in terms of algebraic equations and their similarity with flux vacua has also been established [16, 17].

Although the non-supersymmetric extremal black holes studied in [15] shared many features with their supersymmetric counterparts, there is one striking difference. The black holes preserving supersymmetry are stable. However, for the non-supersymmetric solution, it turns out that, out of the n complex scalar fields only $(n+1)$ real scalar fields are massive and the remaining $(n-1)$ real scalars have vanishing quadratic terms at the attractor point. The cubic terms are absent. And hence the attractors become stable or unstable depending on whether the quartic terms become positive-definite or not [18]. In some cases, as for instance in the STU model, the potential in these $(n-1)$ real scalars remain exactly flat.

The appearance of these massless fields is because only leading order terms in the string theory are considered. In the present work we explore the possibility of lifting these flat directions by including quantum corrections. In some of the simple cases, the exact prepotential for string theory on various one and two-parameter families of Calabi-Yau models, (including the instanton effects), has been computed by using mirror symmetry [19–24]. We examine one of the simplest two-parameter Calabi-Yau manifolds considered in these papers and study the behavior of non-supersymmetric attractors arising from IIA compactification on it. We then generalize the analysis to compactification on arbitrary Calabi-

Yau manifolds, but for simplicity, we retain only the instanton contributions. We show that the instanton contribution to the prepotential lifts these massless directions.

This paper is organized as follows: In §2 we give an introduction to non-supersymmetric attractors. First we discuss non-supersymmetric attractors in the context of $N = 2$ theories. We summarize the basic formalism in terms of the effective potential, including the conditions associated with the non-supersymmetric attractors. There we outline the construction of the effective potential in terms of the super-potential and Kähler potential and the computation of the black hole entropy in terms of the scalars and charges of the black hole. We then consider non-supersymmetric attractors in type IIA string theory compactified on a Calabi-Yau three-fold and we discuss the computation of supersymmetric and non-supersymmetric black hole solution from the from the effective potential. We compute the mass matrix and its spectrum for the D0-D4 system.

In §3 we discuss a simple model of non-supersymmetric attractors for type IIA string theory compactified in a simple two-parameter Calabi-Yau manifold. In Section 4 we consider non-supersymmetric attractors with an instanton correction in type IIA string theory compactified on an arbitrary Calabi-Yau three-fold. Here we construct the effective potential in the context of instantons. We obtain both the supersymmetric as well as the non-supersymmetric attractor solutions for the extremal black hole. We also construct the mass matrix by computing the second derivatives of the effective potential. We find the spectrum of the mass matrix in simple models such as the STU model. We show that with a suitable choice of instanton parameters we will can lift the flat directions of the mass matrix using numerical analysis and degenerate perturbation theory. Finally we conclude by summarizing the main results of the paper followed by an outline of future work.

2 Non-supersymmetric attractors

In this section we will give a brief introduction to attractor mechanism in four dimensional $N = 2$ supergravity theory arising from compactification of string theory on Calabi-Yau three-folds. The supersymmetric attractors were first studied by Ferrara et al [1] by solving the spinor conditions. Subsequently the Euler-Lagrange equations were analyzed in detail and an effective one-dimensional description were given [5]. A detail analysis of the non-supersymmetric attractors were studied much later [14]. In the following we will first discuss the effective potential in general $N = 2$ supergravity theory [5] followed by a review of some of the known results on attractor mechanism in string theory which will be useful for the subsequent discussions.

The bosonic part of the Lagrangian for the $N = 2$ supergravity coupled to N vector multiplets has the following form [5]:

$$\mathcal{L} = -\frac{1}{2}R + \frac{1}{2}g_{a\bar{b}}\partial_\mu\phi^a\partial_\nu\bar{\phi}^{\bar{b}}h^{\mu\nu} - \frac{1}{4}\mu_{ab}\mathcal{F}^a_{\mu\nu}\mathcal{F}^b_{\lambda\rho}h^{\mu\lambda}h^{\nu\rho} - \frac{1}{4}\nu_{ab}\mathcal{F}^a_{\mu\nu} * \mathcal{F}^b_{\lambda\rho}h^{\mu\lambda}h^{\nu\rho} \ . \tag{2.1}$$

Here $g_{a\bar{b}}$ is the moduli space metric, $h_{\mu\nu}$ is the metric on the four dimensional space-time, R is the corresponding Ricci scalar, X^a are the scalar fields and $A^a_\mu, a = \{0, 1, \cdots n\}$, correspond to the Graviphoton and the vector fields arising from the vector multiplets, $\mathcal{F}^a_{\mu\nu}$ are the corresponding field strengths. The gauge couplings μ_{ab} and ν_{ab} depend on the scalar fields X^a. For $N = 2$ supergravity theories, both these matrices and the moduli space metric $g_{a\bar{b}}$ are completely determined from a prepotential.

We are interested in studying static, spherically symmetric black holes. The space-time metric for such black holes has the form:

$$ds^2 = e^{2U}dt^2 - e^{-2U}\gamma_{mn}dx^m dx^n \ . \tag{2.2}$$

For such a metric, the problem of finding the black hole solution can be reduced to an effective one dimensional theory with a constraint. The effective potential for this one dimensional theory is given by [5]:

$$V \;=\; e^K\left[g^{a\bar{b}}\nabla_a W\left(\nabla_b W\right)^* + |W|^2\right] \tag{2.3}$$

where $\nabla_a W = \partial_a W + \partial_a K W$. Here K is the Kähler potential in the moduli space and W is the superpotential. In terms of the $N = 2$ prepotential (and the scalar fields X^a), they have the following form:

$$K \;=\; -\ln \mathrm{Im}\left(\sum_{a=0}^{N}\bar{X}^{\bar{a}}\partial_a F(X)\right)$$

$$W = \sum_{a=0}^{N} \Big(q_a X^a - p^a \partial_a F \Big) \ . \tag{2.4}$$

Here q_a and p^a are the electric and magnetic charges respectively. The condition for the existence of a regular horizon where the scalar fields ϕ^a take finite values is given by [5, 14]:

$$\partial_a V = 0 \ \text{ at the horizon.} \tag{2.5}$$

For the effective potential (2.3), this takes the form:

$$\partial_a V = e^K \Big(g^{b\bar{c}} \nabla_a \nabla_b W \overline{\nabla_c W} + 2 \nabla_a W \overline{W} + \partial_a g^{b\bar{c}} \nabla_b W \overline{\nabla_c W} \Big) = 0 \ . \tag{2.6}$$

This equation is trivially solved if $\nabla_a W = 0$. This condition extremises the central charge $Z = e^{K/2} W$ and corresponds to the supersymmetric attractor. However, Eq.(2.6) admits more general solutions which need not extremize Z. Such solutions correspond to the non-supersymmetric attractors. Unlike the supersymmetric solutions, they are generically not stable. Their stability depends on the mass matrix $M_{ij} = (1/2)\ \partial_i \partial_j V$. The nonsupersymmetric attractors are stable if the corresponding mass matrices M_{ij} admit positive eigenvalues [14].

Let us now focus our discussion to $N = 2$ supergravity theories arising from type IIA string theory compactified on a Calabi-Yau manifold. For this case, the leading order term in the prepotential is given by:

$$F = D_{abc} \frac{X^a X^b X^c}{X^0} \ , \tag{2.7}$$

where the triple intersection numbers D_{abc} are completely determined by the topology of the Calabi-Yau manifold:

$$D_{abc} = \frac{1}{6} \int_M \alpha_a \wedge \alpha_b \wedge \alpha_c \ . \tag{2.8}$$

Here the two forms α_a form a basis of the cohomology $H^2(M; \mathbb{Z})$ of the Calabi-Yau manifold M, over which the integration is carried out.

Let us first consider supersymmetric attractors. They were first derived in [25]. In this paper we will focus only on black hole configurations carrying $D0-D4$ charges. For such configurations, the Kähler potential K and the superpotential W are respectively given by:

$$K = - \ln \left[-i D_{abc} \left(\frac{X^a}{X^0} - \frac{\bar{X}^a}{\bar{X}^0} \right) \left(\frac{X^b}{X^0} - \frac{\bar{X}^b}{\bar{X}^0} \right) \left(\frac{X^c}{X^0} - \frac{\bar{X}^c}{\bar{X}^0} \right) \right] \ , \tag{2.9}$$

$$W = q_0 X^0 - 3\frac{D_{ab} X^a X^b}{X^0} \ . \tag{2.10}$$

Here we have introduced the matrix $D_{ab} \equiv D_{abc} p^c$. For later use, we define $D_a = D_{ab} p^b$ and $D = D_a p^a$. The matrix D^{ab} is the inverse of D_{ab}. The supersymmetric solution obtained by setting $\nabla_a W = 0$, is given by

$$\frac{X^a}{X^0} = i p^a \sqrt{\frac{q_0}{D}} \ . \tag{2.11}$$

The entropy of the black hole is given by $S = 2\pi\sqrt{q_0 D}$. Clearly, this is a valid solution only when $q_0 D$ is positive.

The non-supersymmetric attractors for this system has been found [15]. Interestingly, the non-supersymmetric solution exists in the domain of the charge lattice where the supersymmetric solution does not exist, i.e. when $q_0 D$ is negative. They are obtained by solving the condition $\partial_a V = 0$ such that $\nabla_a W \neq 0$. For the $D0 - D4$ system the solution is given by:

$$\frac{X^a}{X^0} = i p^a \sqrt{-\frac{q_0}{D}} \ . \tag{2.12}$$

Unlike the supersymmetric solutions, the non-supersymmetric ones are not guaranteed to be stable. For every charge configuration, we explicitly need to check if the mass matrix is positive definite. For the $D0 - D4$ system the mass matrix has the following form [15]:

$$M = 24 D {t_0}^2 \left(\left(\frac{3 D_a D_b}{D} - D_{ab} \right) \otimes \mathbf{I} + D_{ab} \otimes \sigma^3 \right) \tag{2.13}$$

It can be easily seen that the mass matrix has $(n+1)$ positive and $(n-1)$ zero eigenvalues. One can explicitly compute terms beyond the quadratic order in the effective potential [18]. For the $D0 - D4$ system the cubic term vanishes and the quartic term is given by the difference of two positive definite terms. Thus, depending on the intersection numbers the quartic term can either be positive definite or negative definite. It can even give rise to flat directions. In this paper we are interested to study the effect of the sub-leading terms in the prepotential on the eigenvalues of the mass matrix. In the following section we will consider an explicit example where the prepotential is known exactly to all orders and discuss attractor mechanism in this example.

3 Attractors in a two-parameter model

In the previous section, we have noticed that the leading term in the effective potential for the $D0-D4$ system admits critical points with $(n+1)$ massive and $(n-1)$ massless modes. It would be interesting to see if these $(n-1)$ massless modes are lifted by sub-leading corrections to the prepotential. The expression for the prepotential has been computed exactly in some one and two-parameter Calabi-Yau models by using mirror symmetry [19–24]. Although the one-parameter models provide the simplest systems to consider, the non-supersymmetric attractors in this case are completely stable. For models with three or more parameters the analysis becomes more cumbersome [26]. Hence, to study the stability, we will focus our attention to the two-parameter case and study the effect of sub-leading terms in the prepotential on the critical points of the effective potential.

In this section we will consider one such example where the Calabi-Yau manifold is a degree eight hypersurface in the weighted projective space $\mathbb{P}_4^{(1,1,2,2,2)}$. The prepotential for this model has been computed exactly [19] and has the following form:

$$\begin{aligned} F &= -\frac{1}{6X^0}\Big(8(X^1)^3+12(X^1)^2X^2\Big)-2X^1X^2-\frac{11}{3}X^0X^1-X^0X^2+\zeta(X^0)^2 \\ &- \frac{640}{(2\pi i)^3}{X^0}^2 e^{\frac{2\pi i X^1}{X^0}}-\frac{4}{(2\pi i)^3}{X^0}^2 e^{\frac{2\pi i X^2}{X^0}}+\cdots \quad . \end{aligned} \tag{3.1}$$

Let us consider the first line in the above equation. The first term is determined by explicitly solving the Picard-Fuchs equation for the periods. The coefficients of all other monomials except the last term, are determined by requiring the symplectic invariance of the monodromy matrix about the boundary divisors [19]. The coefficient ζ in the last term is not determined this way. However the effective potential does not depend on ζ and hence now on we will ignore this term. The second line in Eq.(3.1) arises due to the non-perturbative contribution to the prepotential and is determined by using mirror symmetry. In this section we will ignore these non-perturbative contributions and see the effect of perturbative corrections. In the next section we will consider instanton contributions in a more general context and discuss their implications.

Let us first consider the supersymmetric solution for the $D0-D4$ configuration ignoring both perturbative as well as non-perturbative corrections. The leading term in the prepotential is:

$$F_0 = -\frac{1}{6X^0}\Big(8(X^1)^3 + 12(X^1)^2X^2\Big)\ . \tag{3.2}$$

The supersymmetric attractor is given by

$$\frac{X^1}{X^0} = i\sqrt{\frac{-3q_0}{2\,(2p^1+3p^2)}} \tag{3.3}$$

$$\frac{X^2}{X^0} = i\frac{p^2}{p^1}\sqrt{\frac{-3q_0}{2\,(2p^1+3p^2)}} \tag{3.4}$$

where as the non-supersymmetric attractor is given by

$$\frac{X^1}{X^0} = i\sqrt{\frac{3q_0}{2\,(2p^1+3p^2)}} \tag{3.5}$$

$$\frac{X^2}{X^0} = i\frac{p^2}{p^1}\sqrt{\frac{3q_0}{2\,(2p^1+3p^2)}} \tag{3.6}$$

It is important to note that the supersymmetric solution exists if and only if $q_0(2p^1 + 3p^2) < 0$ where as the non-supersymmetric solution exists only when $q_0(2p^1 + 3p^2) > 0$.

Let us now consider sub-leading corrections to the prepotential ignoring the non-perturbative part:

$$F = -\frac{1}{6X^0}\Big(8(X^1)^3 + 12(X^1)^2X^2\Big) - 2X^1X^2 - \frac{11}{3}X^0X^1 - X^0X^2 + \zeta X^{0^2} \tag{3.7}$$

We are interested in studying both supersymmetric as well as non-supersymmetric solutions for the $D0 - D4$ system with this prepotential. Let us first consider the supersymmetric case. The superpotential and the Kähler potential can be evaluated using the formulae (2.4). Defining $x_1 = X^1/X^0$ and $x_2 = X^2/X^0$ and choosing the gauge $X^0 = 1$ we find

$$\begin{aligned} W &= q_0 + p^1\left(4x_1(x_1+x_2) + 2x_2 + \frac{11}{3}\right) + p^2(2{x_1}^2 + 2x_1 + 1) \\ K &= -\ln\left(\frac{2i}{3}(x_1-\bar{x}_1)^2\left(2(x_1-\bar{x}_1) + 3(x_2-\bar{x}_2)\right)\right) \end{aligned} \tag{3.8}$$

The condition for supersymmetry is given by the two complex equations $\nabla_1 W = 0 = \nabla_2 W$. Using the explicit expression for W and K from Eq.(3.8), and setting

$x_1 = u_1 + iu_2, x_2 = v_1 + iv_2$, we find, for $u_2 \neq 0$ and $(2u_2 + 3v_2) \neq 0$, the real and imaginary parts of both these equations are respectively given by

$$
\begin{aligned}
u_2^2\Big(p^2(1+2u_1) + 2p^1(2u_1+v_1)\Big) + 3p^1 v_2(1+2u_1)(u_2+v_2) &= 0\ , \quad (3.9)\\
3(p^1 v_2 - p^2 u_2)(1+2u_1) + 2p^1 u_2(2-2u_1-3v_1) &= 0\ , \quad (3.10)\\
4u_2^2 v_2(p^1+p^2) + \Big(3q_0 + p^1\{11 + 4u_2^2 + 6v_1 + 12u_1(u_1+v_1)\}& \\
+p^2\{3 + 6u_1(1+u_1) + 2u_2^2\}\Big)(u_2+v_2) &= 0\ , \quad (3.11)\\
3q_0 + p^1\Big(11 + 6v_1 + 12u_1(u_1+v_1) + 4u_2(u_2+3v_2)\Big)& \\
+3p^2\Big(1 + 2u_1(1+u_1) - 2u_2^2\Big) &= 0\ . \quad (3.12)
\end{aligned}
$$

Surprisingly the above set of equations do not admit any consistent solution for generic values of the $D0 - D4$ charges. This can easily be seen as follows. For $u_2 \neq 0$ and $2u_2 + 3v_2 \neq 0$, Eqs.(3.11) and (3.12) admit a unique solution for v_1 and v_2 in terms of u_1, u_2 and the charges q_0, p^1, p^2, which is given by

$$
\begin{aligned}
v_1 &= \frac{-1}{6p^2(1+2u_1)}\Big(3q_0 + 3p^2\Big(1 + 2u_1(1+u_1) + 2u_2^2\Big) + p^1\Big(11 + 12u_1^2 + 4u_2^2\Big)\Big)\\
v_2 &= \frac{p^2}{p^1}u_2\ . \qquad (3.13)
\end{aligned}
$$

We can substitute these expressions for v_1 and v_2 in Eqs.(3.9) and (3.10). For $(1 + 2u_1) \neq 0$ we get

$$
\begin{aligned}
3(q_0+p^2) + 6p^2(u_1 + u_1^2 + u_2^2) + p^1\Big(15 + 4u_1(1+u_1) + 4u_2^2\Big) &= 0\ , \quad (3.14)\\
9(p^2)^2(1+2u_1)^2 + (p^1)^2\Big(-11 + 12u_1(1+u_1) - 4u_2^2\Big) - 3q_0 p^1& \\
+p^1 p^2\Big(9 + 42u_1(1+u_1) - 6u_2^2\Big) &= 0\ . \quad (3.15)
\end{aligned}
$$

It can easily be verified that all the solutions for the above set of equations has $u_1 = -1/2$ in contradiction to our assumption $(1+2u_1) \neq 0$. We can try to solve Eqs.(3.9)-(3.12) eliminating different set of variables and so on. However we will reach the same conclusion, i.e., these set of equations do not admit any solution for generic values of the $D0 - D4$ charges.

Though we do not have supersymmetric black holes in this very simple model for generic values of the charges, we do have consistent solutions when the black hole charges obey certain constraint. To see this, consider the ansatz $x_1 = p^1 t$

and $x_2 = p^2 t$. For this ansatz, the susy conditions $\nabla_1 W = 0 = \nabla_2 W$ give two complex equations involving the complex variable t:

$$\begin{aligned} 11(p^1)^2 + 14p^1p^2 + 3(p^2)^2 + 3p^1q_0 + 3p^2q_0 + 10(p^1)^2p^2t & \\ +9p^1(p^2)^2t4(p^1)^4t^2 + 10(p^1)^3p^2t^2 + 6(p^1)^2p^{22}t^2 + 2(p^1)^2p^2\bar{t} & \\ +3p^1(p^2)^2\bar{t} + 8(p^1)^4|t|^2 + 20(p^1)^3p^2|t|^2 + 12(p^1)^2(p^2)^2|t|^2 &= 0 \quad (3.16) \\ 11p^1 + 3p^2 + 3q_0 - 4(p^1)^2t + 6p^1p^2t + 4(p^1)^3t^2 & \\ +6(p^1)^2p^2t^2 + 4(p^1)^2\bar{t} + 6p^1p^2\bar{t} + 8(p^1)^3|t|^2 + 12(p^1)^2p^2|t|^2 &= 0 \quad (3.17) \end{aligned}$$

It is straightforward to check that, both these equations coincide if and only if $p^2 = -2p^1$. In this case the equation of motion has the following simple form:

$$5p^1 + 3q_0 - 8(p^1)^2(2t + \bar{t}) - 8(p^1)^3t(t + 2\bar{t}) = 0 \quad (3.18)$$

It is now straightforward to solve the above equation for t. For the supersymmetric solution with the ansatz $x_a = p^a t$, the expression for t is given by

$$t = -\frac{1}{2p^1} - i\frac{1}{2\sqrt{2}p^1}\sqrt{11 + \frac{3q_0}{p^1}} \,. \quad (3.19)$$

The entropy of the supersymmetric black hole can be evaluated by using the equation

$$S = \pi e^K |W|^2 \quad (3.20)$$

and is given by

$$S = \frac{4}{3}(p^1)^2\pi\sqrt{22 + \frac{6q_0}{p^1}} \,. \quad (3.21)$$

Before we turn our attention to the non-supersymmetric attractors, it is worth emphasizing a few important points. First, the solution exists only if $(11 + 3q_0/p^1) > 0$. However this is not so different from the condition $q_0(2p^1 + 3p^2) < 0$ for the existence of the supersymmetric solution when we consider only the leading term in the prepotential. In fact both the conditions are identical in the limit $|q_0| \gg |p^1|$ provided $p^2 = -2p^1$. As expected, the exact solution (3.18) coincides with the leading order solution (3.4) up to terms of order $(1/p^1)$. What is more surprising is the condition $p^2 = -2p^1$ imposed on the $D4$-brane charges for the existence of supersymmetric solution when we consider sub-leading corrections to the prepotential. Naively one would expect that the sub-leading terms will change the supersymmetric solution only in a small way. However, what we observe is that the solution ceases to exist for generic values of $D4$-brane charges. We do not know if this is an artifact of the toy example we are considering in this section. The curvature corrections to the $N = 2$ supergravity Lagrangian (2.1) might restore the solutions for all values of the charges. Clearly, we need further investigation to understand this issue.

Now we will discuss non-supersymmetric attractors in the presence of sub-leading terms in the prepotential. In this case we need to extremize the effective potential to find the solution. The effective potential for our case has been derived in §A.1. We find

$$\begin{aligned} V &= \Big[\Big(6{q_0}^2 + 44p^1q_0 + 12p^2q_0 + (242/3)(p^1)^2 + 6(p^2)^2 + 44p^1p^2\Big) \\ &+ \Big(24p^1q_0 + 12p^2q_0 + 96(p^1)^2 + 30(p^2)^2 + 60p^1p^2\Big)|x_1|^2 + 24(p^1)^2|x_2|^2 \\ &+ \Big(64(p^1)^2 + 24(p^2)^2 + 64p^1p^2\Big)|x_1|^4 + 72(p^1)^2|x_1x_2|^2 + \Big(12p^1q_0x_2\bar{x_1} \\ &+ 12p^1q_0x_1^2 + 6p^2q_0x_1^2 + 12p^2q_0x_1 + 12p^1q_0x_2 + 12p^1q_0x_1x_2 + 8(p^1)^2x_2{\bar{x_1}}^3 \\ &+ 16(p^1)^2x_1^3\bar{x_1} + 16p^1p^2x_1^3\bar{x_1} + 24(p^2)^2x_1^2\bar{x_1} + 40p^1p^2x_1^2\bar{x_1} + 52(p^1)^2x_2\bar{x_1} \\ &+ 48p^1p^2x_1x_2\bar{x_1} + 12(p^1)^2x_1^2x_2\bar{x_2} + 48(p^1)^2x_1x_2\bar{x_2} + 8p^1p^2x_1^3 + 40(p^1)^2x_1^2 \\ &+ 9(p^2)^2x_1^2 + 38p^1p^2x_1^2 + 12(p^2)^2x_1 + 44p^1p^2x_1 + 44(p^1)^2x_2 + 12p^1p^2x_1^2x_2 \\ &+ 32(p^1)^2x_2{\bar{x_1}}^2 + 12p^1p^2x_2{\bar{x_1}}^2 + 64(p^1)^2x_1x_2{\bar{x_1}}^2 + 24p^1p^2x_1x_2{\bar{x_1}}^2 \\ &+ 24(p^1)^2x_1^2x_2\bar{x_1} + 24p^1p^2x_1^2x_2\bar{x_1} + 24p^1p^2x_2\bar{x_1} + 16(p^1)^2x_1x_2\bar{x_1} \\ &+ 12p^1p^2x_2 + 36(p^1)^2x_1x_2 + 24p^1p^2x_1x_2 + C.C.\Big)\Big] \\ &\times\frac{-i}{(x_1-\bar{x_1})^2(2(x_1-\bar{x_1})+3(x_2-\bar{x_2}))} \ . \end{aligned} \tag{3.22}$$

The equations of motion are found by extremizing the above potential. It is interesting to note that, in this case also both the equations $\partial_1 V = 0$ and $\partial_2 V = 0$ coincide only when the $D4$-brane charges satisfy $p^2 = -2p^1$. After imposing this constraint we find

$$\begin{aligned} &30p^1q_0 + 9{q_0}^2 + 32(p^1)^6\bar{t}^2(7t^2 + 10|t|^2 + \bar{t}^2) + 32(p^1)^5\bar{t}(7t^2 + 22|t|^2 + 7\bar{t}^2) \\ &\quad + 4(p^1)^4(3t^2 + 50|t|^2 + 31\bar{t}^2) + (p^1)^2(25 - 48q_0(t + 2\bar{t})) - 4(p^1)^3(3q_0t^2 \\ &\qquad + 5\bar{t}(8 + 3q_0\bar{t}) + 2t(10 + 9q_0\bar{t})) = 0 \ . \end{aligned} \tag{3.23}$$

Solving this equation, we find, for the non-supersymmetric black holes

$$t = -\frac{1}{2p^1} - \frac{i}{2\sqrt{2}p^1}\sqrt{-11 - \frac{3q_0}{p^1}} \ . \tag{3.24}$$

The entropy of the non-supersymmetric black hole is found to be

$$S_{Nonsusy} = \frac{4}{3}(p^1)^2\pi\sqrt{-22 - \frac{6q_0}{p^1}} \ . \tag{3.25}$$

The results are pretty much similar to the ones for the supersymmetric black hole attractors. The black hole solution does not exist for generic charge configuration. It exists only on the two dimensional sub-lattice obtained by imposing the condition $p^2 = -2p^1$ of the charge lattice spanned by q_0, p^1, p^2. In this sub-lattice the supersymmetric solution exists when $(11 + 3p_0/p^1) > 0$ where as non-supersymmetric solution exists for $(11 + 3q_0/p^1) < 0$. We need to check the stability of the non-supersymmetric black hole solution. The mass matrix for the black hole can be obtained by using the formula

$$M = \partial_a\partial_{\bar{b}}V \otimes \mathbf{I} + \Re(\partial_a\partial_b V) \otimes \sigma^{\mathbf{3}} - 2\Im(\partial_a\partial_b V) \otimes \sigma^{\mathbf{1}} \ . \tag{3.26}$$

Each of the terms are evaluated in §A.1. The mass matrix is found to be

$$M = \frac{\sqrt{2}(p^1)^2}{\sqrt{-11 - \frac{3q_0}{p^1}}} \begin{pmatrix} 12 & 0 & -6 & 0 \\ 0 & 12 & 0 & 2 \\ -6 & 0 & 3 & 0 \\ 0 & 2 & 0 & 3 \end{pmatrix} \ . \tag{3.27}$$

It is straightforward to diagonalize it. The eigenvalues of the mass matrix are given by

$$\lambda = \frac{\sqrt{2}(p^1)^2}{\sqrt{-11 - \frac{3q_0}{p^1}}} \Big(15, \frac{1}{2}\big(15 + \sqrt{97}\big), \frac{1}{2}\big(15 - \sqrt{97}\big), 0\Big) \ . \tag{3.28}$$

Interestingly we observe that the zero-mode still survives. Thus, the perturbative corrections to the prepotential restrict the allowed black hole solutions to a sub-space of the charge lattice. However the issue of stability for the non-supersymmetric attractors still remains to be shorted out.

4 Type IIA at large volume with instanton corrections

In the previous section we considered $D0-D4$ configuration in a two-parameter Calabi-Yau model and found supersymmetric as well as non-supersymmetric attractors. In both the case, the sub-leading corrections to the prepotential forced the solution to exist only in a sub-lattice of the charge lattice for the $D0-D4$ system. When restricted to this sub-lattice, the exact solution in the presence of the sub-leading terms differ from the leading solution by terms of order $O(1/p^1)$. For the non-supersymmetric critical point, the massless mode did not get lifted by these corrections. We would like to study the attractor solution in a more general context in stead of restricting ourselves to a specific two-parameter Calabi-Yau manifold. In the general case also the perturbative corrections to the prepotential will impose a few constraints on the charges. Apart from the above restriction, we do not expect any drastic change in the qualitative behavior of the attractor solution due to these perturbative corrections. In particular, in the case of the non-supersymmetric solution, we expect a few massless modes surviving these corrections. Hence, in order to lift these massless directions, we would like to incorporate instanton corrections to the prepotential. In the presence of the instanton term the computation for the effective potential and the mass matrix is extremely tedious. Hence we will restrict ourselves to the simplest non-trivial case where the prepotential has the form

$$F = D_{abc}\frac{X^a X^b X^c}{X^0} + A(X^0)^2 e^{C_a X^a/X^0} \ . \tag{4.1}$$

Here the coefficients A and C_a in the pre-factor as well as in the exponent of the instanton term are constants which will depend on the details on the Calabi-Yau manifold under consideration. Throughout this section we will assume that the contribution due to the exponential term is small and we will ignore higher powers of the exponential term.

We will now compute the expression for the superpotential and the Kähler potential for the system. For the $D0-D4$ configuration the superpotential is given by

$$W = q_0 X^0 - 3\frac{D_{ab} X^a X^b}{X^0} - \frac{A C_a p^a}{X^0} e^{C_b X^b/X^0} \ . \tag{4.2}$$

The Kähler potential for the system is found to have the form

$$
\begin{aligned}
K &= -\ln\Big[- iD_{abc}\left(\frac{X^a}{X^0}-\frac{\bar{X}^a}{\bar{X}^0}\right)\left(\frac{X^b}{X^0}-\frac{\bar{X}^b}{\bar{X}^0}\right)\left(\frac{X^c}{X^0}-\frac{\bar{X}^c}{\bar{X}^0}\right) \\
&- i\left(\frac{X^a}{X^0}-\frac{\bar{X}^a}{\bar{X}^0}\right)\left(A\frac{\bar{X}^0}{X^0}C_a e^{C_b X^b/X^0}+\bar{A}\frac{X^0}{\bar{X}^0}\bar{C}_a e^{\bar{C}_b\bar{X}^b/\bar{X}^0}\right) \\
&+ 2iX^0\bar{X}^0\left(Ae^{C_bX^b/X^0}-\bar{A}e^{\bar{C}_b\bar{X}^b/\bar{X}^0}\right)\Big] \; . \qquad (4.3)
\end{aligned}
$$

From now on, we will introduce $x^a = X^a/X^0$ and set the gauge $X^0 = 1$. With this choice, the superpotential and Kähler potential reads

$$
W = q_0 - 3D_{ab}x^a x^b - AC_a p^a e^{C_b x^b} \qquad (4.4)
$$

$$
\begin{aligned}
K &= -\ln\Big[- i\Big(D_{abc}\left(x^a-\bar{x}^a\right)\left(x^b-\bar{x}^b\right)\left(x^c-\bar{x}^c\right) \\
&+ \left(x^a-\bar{x}^a\right)\left(AC_a e^{C_b x^b}+\bar{A}\bar{C}_a e^{\bar{C}_b\bar{x}^b}\right)+2\left(-Ae^{C_b x^b}+\bar{A}e^{\bar{C}_b\bar{x}^b}\right)\Big)\Big] \; . \qquad (4.5)
\end{aligned}
$$

For convenience, we introduce the constant $T = C_a p^a$ and the function $k(x,\bar{x}) = C_a\left(x^a - \bar{x}^a\right)$. We will often denote k for the function $k(x,\bar{x})$. In addition, we define

$$
M = D_{abc}\left(x^a-\bar{x}^a\right)\left(x^b-\bar{x}^b\right)\left(x^c-\bar{x}^c\right) \qquad (4.6)
$$

$$
L = A(k-2)e^{C_b x^b} - \bar{A}(\bar{k}-2)e^{\bar{C}_b\bar{x}^b} \; . \qquad (4.7)
$$

In terms of these quantities the superpotential and the Kähler potential have the simple form

$$
W = q_0 - 3D_{ab}x^a x^b - ATe^{C_b x^b} \qquad (4.8)
$$

$$
K = -\ln\left[-i\left(M+L\right)\right] \; . \qquad (4.9)
$$

4.1 Supersymmetric solution

In this subsection we will consider the supersymmetric solution for the system. Hence we need to solve the equation

$$
\nabla_a W = \partial_a W + \partial_a K W = 0 \; . \qquad (4.10)
$$

Substituting the expression for W and K we get

$$
\begin{aligned}
&- \frac{1}{M+L}\Big[6MD_{ab}x^b + 3M_a W^0 + \Big(MTC_a - 3M_a T + C_a(k-1)W^0 \\
&+ 6D_{ab}x^b(k-2)\Big)Ae^{C_b x^b} + \Big(\bar{C}_a W^0 - 6D_{ab}x^b(\bar{k}-2)\Big)\bar{A}e^{\bar{C}_b\bar{x}^b}\Big] = 0 \; , \qquad (4.11)
\end{aligned}
$$

where

$$W^0 = q_0 - 3D_{ab}x^a x^b \ , \tag{4.12}$$

is the leading order superpotential for the $D0 - D4$ system. In the absence of the instanton contribution the solution for supersymmetric attractor is given by

$$x^a = ip^a \sqrt{\frac{q_0}{D}} \ . \tag{4.13}$$

We expect the solution of Eq.(4.11) for x^a to differ from the above by a term of order $e^{C_a x^a}$. Hence we set the ansatz

$$x^a = i\sqrt{\frac{q_0}{D}} p^a + \tilde{x}^a \tag{4.14}$$

to solve Eq.(4.11) and keep terms up to order $\tilde{x}^a$. It is convenient to consider $p^a \nabla_a W = 0$. After simplification we get,

$$6iq_0\sqrt{\frac{q_0}{D}} D_c \bar{\tilde{x}}^c + Ae^{C_b x^b}\left(3q_0 T + 3iD\sqrt{\frac{q_0}{D}}\right) + \left(2q_0 \bar{T} + 3iD\sqrt{\frac{q_0}{D}}\right) = 0 \ , \tag{4.15}$$

which gives

$$D_c \bar{\tilde{x}}^c = \frac{1}{6i}\sqrt{\frac{D}{q_0}}\left(\left(2\bar{T} + 3i\sqrt{\frac{D}{q_0}}\right)\bar{A}e^{\bar{C}_b \bar{x}_0^b} - \left(3T + 3i\sqrt{\frac{D}{q_0}}\right)Ae^{C_b x_0^b}\right) \ . \tag{4.16}$$

The entropy of the supersymmetric black hole is given by

$$S_{BH} = \pi e^K |W|^2|_{\phi_{i0}} \tag{4.17}$$

Note that

$$|W|^2|_{\phi_{i0}} = 16q_0^2 + 24q_0 i\sqrt{\frac{q_0}{D}}\left(D_a\bar{\tilde{x}}^a - D_a\tilde{x}^a\right) - 4q_0\left(ATe^{C_b x_0^b} + \bar{A}\bar{T}e^{\bar{C}_b \bar{x}_0^b}\right) \tag{4.18}$$

and

$$e^K \ |_{\phi_{i0}} = \frac{1}{8D{t_0}^3} + \frac{1}{16D^2{t_0}^6}\Re\left[(2Tt_0 + 7i)Ae^{C_b {x_0}^b}\right] \ . \tag{4.19}$$

Substituting for $(D_a\bar{\tilde{x}}^a - D_a\tilde{x}^a)$ in $|W|^2$, we get

$$|W|^2|_{\phi_{i0}} = 16q_0^2 - \Re\left[(2T + \frac{6i}{t_0})Ae^{C_b {x_0}^b}\right] \ . \tag{4.20}$$

After simplifying a bit, we get

$$S_{BH} = 2\pi\sqrt{q_0 D} - \frac{\pi}{{t_0}^2}\Im\left[Ae^{C_b {x_0}^b}\right] \ , \tag{4.21}$$

where $t_0 = \sqrt{\frac{q_0}{D}}$.

4.2 Effective potential with instanton corrections

We will now consider the non-supersymmetric solution for the $D0-D4$ system keeping the instanton contribution to the prepotential. For this purpose, we need to consider the effective potential

$$V = e^K \left[g^{a\bar{b}} \nabla_a W \left(\nabla_b W \right)^* + |W|^2 \right] \tag{4.22}$$

and find it's critical points for which $\nabla_a W \neq 0$. In the following we will evaluate the effective potential for the system and in the next subsection we will extremize it to find the non-supersymmetric critical points.

Let us first evaluate the metric $g_{a\bar{b}}$ on the moduli space.

$$\begin{aligned} g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K &= \frac{1}{(M+L)^2} \Big[- M\partial_{\bar{b}}\partial_a M - M\partial_{\bar{b}}\partial_a L - L\partial_{\bar{b}}\partial_a M \\ &+ \partial_a M \partial_{\bar{b}} M + \partial_a M \partial_{\bar{b}} L + \partial_a L \partial_{\bar{b}} M \Big] \ . \end{aligned} \tag{4.23}$$

Substituting for the expressions inside the bracket we get

$$\begin{aligned} g_{a\bar{b}} &= \frac{1}{(M+L)^2} \Big[6MM_{ab} - 9M_a M_b + \Big(Ae^{C_b x^b} \Big(MC_a C_b + 6M_{ab}(k-2) \\ &- 3M_a C_b - 3M_b C_a (k-1) \Big) + H.C \Big) \Big] \ . \end{aligned} \tag{4.24}$$

The inverse of the metric $g_{a\bar{b}}$ is found to be

$$\begin{aligned} g^{c\bar{d}} &= \frac{(M+L)^2}{6M^2} \Big[MM^{cd} - 3\left(x^c - \bar{x}^c\right)\left(x^d - \bar{x}^d\right) - \Big(\frac{1}{6M} \Big(M^2 M^{qc} M^{pd} C_p C_q \\ &+ 3MM^{cq} C_q (x^d - \bar{x}^d)(2-k) + 3MM^{dq} C_q (x^c - \bar{x}^c)(k+6) \\ &+ 6MM^{cd}(k+2) - 9k(x^c - \bar{x}^c)(x^d - \bar{x}^d)(k+6) \\ &+ 36(x^c - \bar{x}^c)(x^d - \bar{x}^d) \Big) Ae^{C_b x^b} + H.C. \Big) \Big] \ . \end{aligned} \tag{4.25}$$

On the other hand, the covariant derivative of the superpotential is given by

$$\begin{aligned} \nabla_a W &= -\frac{1}{M+L} \Big[6MD_{ab}x^b + 3M_a W^0 + \Big(MTC_a - 3M_a T + C_a (k-1) W^0 \\ &+ 6D_{ab}x^b (k-2) \Big) Ae^{C_b x^b} + \left(\bar{C}_a W^0 - 6D_{ab} x^b (\bar{k}-2) \right) \bar{A} e^{\bar{C}_b \bar{x}^b} \Big] \ . \end{aligned} \tag{4.26}$$

Substituting these terms in the expression for the effective potential and simplifying we get

$$V = Ue^K = e^K \left(U_0 + U_1 Ae^{C_b x^b} + \bar{U}_1 \bar{A} e^{\bar{C}_b \bar{x}^b} \right) \ , \tag{4.27}$$

where

$$
\begin{aligned}
U_0 &= 4|W^0|^2 + 6D_{aq}\left(x^a - \bar{x}^a\right)\left(x^q\bar{W}^0 - \bar{x}^q W^0\right) \\
&+ 6D_{aq}D_{bd}x^q\bar{x}^d\left(MM^{ab} - 3\left(x^a - \bar{x}^a\right)\left(x^b - \bar{x}^b\right)\right)
\end{aligned}
\tag{4.28}
$$

and

$$
\begin{aligned}
U_1 &= \frac{1}{M}\Big(D_{aq}D_{bd}x^q\bar{x}^d\left[-3MM^{pb}C_p\left(x^a - \bar{x}^a\right)(k+6) + 6MM^{ab}(k-6)\right. \\
&+ 3MM^{pa}C_p\left(x^b - \bar{x}^b\right)(k-2) + 9\left(k^2 + 2k + 4\right)\left(x^a - \bar{x}^a\right)\left(x^b - \bar{x}^b\right) \\
&- \left. M^2M^{qa}M^{pb}C_pC_q\right] + D_{bd}\bar{x}^d\left[M^2TM^{pb}C_p - 3MT\left(x^a - \bar{x}^a\right)(k-2)\right. \\
&+ 3\left(k^2 + 6k - 4\right)\big)\left(x^b - \bar{x}^b\right)W^0 - \left. M^{pb}C_pW^0\left(k+10\right)\right] \\
&- D_{aq}x^q\left[MM^{ap}C_p\bar{W}^0\left(k-2\right) + 3\left(k^2 + 2k - 4\right)\left(x^a - \bar{x}^a\right)\bar{W}^0\right] \\
&+ M\bar{W}^0T\left(k-4\right) - \left(k^2 + 6k - 12\right)|W^0|^2\Big)\ .
\end{aligned}
\tag{4.29}
$$

4.3 Non-supersymmetric solution

We will now extremize the effective potential (4.27) to obtain the instanton corrected non-supersymmetric solution. We set the ansatz $x^a = p^a t$ and try to solve the equation of motion with the assumption that the solution will differ from the leading order non-supersymmetric solution $x_0^a = ip^a\sqrt{-q_0/D}$ by $O(e^{C_a x_0^a})$. Thus we assume $t = it_0 + (m + is)$ with $t_0 = \sqrt{-q_0/D}$ and keep terms up to linear order in m and s. The equation of motion is a bit tedious in this case and we list some steps for the derivation of this equation in §B.1. We see that, for the above mentioned ansatz, the equation of motion takes the form:

$$
\begin{aligned}
-192iD^3{t_0}^5(3m - is) + 8D^2{t_0}^4\Big(3T - 2it_0T^2 + 2{t_0}^2T^3\Big)Ae^{C_b{x_0}^b} \\
+8D^2{t_0}^4\Big(9\bar{T} - 4it_0\bar{T}^2\Big)\bar{A}e^{\bar{C}_b{\bar{x}_0}^b} = 0\ .
\end{aligned}
\tag{4.30}
$$

Solving the above equation for m and s we get

$$
m = \frac{1}{36Dt_0}\Im\left(T\big(-3 - 3it_0T + T^2{t_0}^2\big)Ae^{C_b{x_0}^b}\right) \tag{4.31}
$$

and

$$
s = -\frac{1}{12Dt_0}\Re\Big(T\Big(6 + it_0T + {t_0}^2T^2\Big)Ae^{C_b{x_0}^b}\Big)\ . \tag{4.32}
$$

For the above solution, we find

$$
\begin{aligned}
U|_{\phi_{i0}} &= 16D^2{t_0}^4 - Dt_0\Re\Big(\Big(-3i + 7Tt_0 + {t_0}^3T^3\Big)Ae^{C_b{x_0}^b}\Big) \ , && (4.33)\\
e^K|_{\phi_{i0}} &= \frac{1}{8D{t_0}^3} + \frac{1}{32D^2{t_0}^6}\Re\Big(\Big(2i + 8Tt_0 + i{t_0}^2T^2 + {t_0}^3T^3\Big)Ae^{C_b{x_0}^b}\Big) && (4.34)
\end{aligned}
$$

Hence, the entropy of the black hole is given by

$$
S = 2\pi Dt_0 + \frac{\pi}{2{t_0}^2}\Re\Big(\Big(5i + 7Tt_0 + i{t_0}^2T^2\Big)Ae^{C_b{x_0}^b}\Big) \ . \qquad (4.35)
$$

4.4 Mass matrix

In the preceding subsections, we have seen that it is possible to find a consistent instanton corrected attractor solution for both the supersymmetric as well as the non-supersymmetric cases. In order to know the stability of the non-supersymmetric attractor, we need to evaluate the mass matrix

$$
M = \partial_b\partial_{\bar{a}}V \otimes \mathbf{I} + \Re(\partial_b\partial_a V) \otimes \sigma^{\mathbf{3}} - 2\Im(\partial_b\partial_a V) \otimes \sigma^{\mathbf{1}} \qquad (4.36)
$$

at the attractor point. The computations are really cumbersome and we give the detail derivation in §B.2. In summary, various terms of the mass matrix are given by

$$
\begin{aligned}
\partial_{\bar{b}}\partial_a V &= 24D{t_0}^2\Big(\frac{3D_aD_b}{D} - D_{ab}\Big) + \Big(Ae^{c_b{x_0}^b}\Big(\frac{2D_{\alpha\beta}}{t_0}\Big(12i + 6t_0T - 2i{t_0}^2T^2\\
&+ T^3{t_0}^3\Big) + \frac{D_aD_b}{Dt_0}\Big(-i - 43t_0T + 3iT^2{t_0}^2 - 6T^3{t_0}^3\Big) + 2C_aC_b\Big(4iDt_0\\
&- DT{t_0}^2\Big) + 2iDt_0TJ_{ab} - \frac{3D_aC_b}{2}\Big(19 + 2iTt_0\Big) + \frac{3D_bC_a}{2}\Big(7 - 8it_0T\\
&+ 2T^2{t_0}^2)\Big) + H.C.\Big) \ , \qquad (4.37)
\end{aligned}
$$

where $J_{ab} = D_{abs}D^{sp}C_p$. The real and imaginary parts of $(\partial_b\partial_a V)$ are found to be

$$
\begin{aligned}
\Re(\partial_b\partial_a V) &= 24DD_{ab}{t_0}^2 + \Big[Ae^{c_b{x_0}^b}\Big(\frac{6D_{ab}}{t_0}\Big(-i + 3Tt_0 + 2iT^2{t_0}^2\Big)\\
&+ \frac{9D_aD_b}{Dt_0}\Big(i - Tt_0 - iT^2{t_0}^2\Big) + Dt_0C_aC_b\Big(-13i + 7Tt_0 + iT{t_0}^2\Big)\\
&+ \frac{3}{2}\Big(2 + 5it_0T - T^2{t_0}^2\Big)\Big(D_aC_b + D_bC_a\Big)\Big) + C.C.\Big] \qquad (4.38)
\end{aligned}
$$

and

$$\begin{aligned}\Im(\partial_b\partial_a V) &= \Big[Ae^{c_b x_0{}^b}\Big(\frac{4}{3}D_{ab}\Big(-3i+3Tt_0+iT^2t_0{}^2\Big)+18iT\frac{D_aD_b}{D}\\ &+ 2DJ_{ab}\Big(-i+t_0T\Big)+Dt_0C_aC_b\Big(-12-7iTt_0+T^2t_0{}^2\Big)\\ &+ \frac{3}{2}\Big(-3i+3Tt_0+iT^2t_0{}^2\Big)\Big(D_aC_b+D_bC_a\Big)\Big)+C.C.\Big]\ . \end{aligned} \tag{4.39}$$

It is in general quite difficult to diagonalize the above mass matrix. In the following subsection we consider the special case when $n = 3$ when the only non-vanishing component of D_{abc} is given by $D_{123} = 1$. In the absence of the instanton correction, this is the STU model. In the next subsection we will analyze the mass matrix for this special case.

4.5 STU model

Consider the special case, where the number of vector multiplets is $n = 3$ and the only non-vanishing intersection number D_{abc} is D_{123}. For simplicity, let $D_{123} = 1$ and choose $p^1 = p^2 = p^3 = p$. In the absence of the subleading corrections, the mass matrix is diagonalizable and the black hole effective potential has two exactly flat directions. In the following subsection we will show that, both the flat directions can be lifted in the presence of instanton term. For this purpose, consider the matrix elements of various terms appearing in the mass matrix:

$$D_{\alpha\beta} = p\begin{pmatrix} 0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{pmatrix},$$

$$D_\alpha D_\beta = 4p^4\begin{pmatrix} 1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}, \tag{4.40}$$

$$D_\alpha C_\beta = 2p^2\begin{pmatrix} C_1 & C_2 & C_3\\ C_1 & C_2 & C_3\\ C_1 & C_2 & C_3\end{pmatrix}, \tag{4.41}$$

$$J_{\alpha\beta} = \frac{1}{2p}\begin{pmatrix} 0 & C_1+C_2-C_3 & C_1-C_2+C_3\\ C_1+C_2-C_3+ & 0 & -C_1+C_2+C_3\\ C_1-C_2+C_3 & -C_1+C_2+C_3 & 0\end{pmatrix}. \tag{4.42}$$

We will scale out a factor of $(p)^2$ from the mass matrix. We will first diagonalize the mass matrix for the STU model using perturbative methods. Subsequently we will do the calculation numerically as well as find the solution by brute force for some special choice of the parameters.

4.6 First order perturbation correction

In this section we will evaluate the spectrum of the mass matrix for the STU model using first order perturbation theory. We will set the mass matrix M as $M = M_0 + M_1$ where M_0 corresponds to the mass matrix in the absence of instanton term and M_1 is the correction piece. The matrix M_0 has the simple form:

$$M_0 = 288 t_0^2 \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} . \tag{4.43}$$

The eigen values of the matrix are given by

$$288 t_0^2 \{0, 0, 1, 1, 1, 3\} \tag{4.44}$$

where $t_0 = \sqrt{-q_0/6p}$. The normalized eigenvectors are given by

$$\begin{aligned} |\psi_i\rangle \;\; &= \;\; \left\{-\frac{1}{\sqrt{2}}, 0, 0, 0, \frac{1}{\sqrt{2}}, 0\right\}, \left\{-\frac{1}{\sqrt{6}}, 0, \sqrt{\frac{2}{3}}, 0, -\frac{1}{\sqrt{6}}, 0\right\}, \{0, 0, 0, 0, 0, 1\}, \\ & \quad , \{0, 0, 0, 1, 0, 0\}\{0, 1, 0, 0, 0, 0\}, \left\{\frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, 0, \frac{1}{\sqrt{3}}, 0\right\} . \end{aligned} \tag{4.45}$$

The expression for the matrix elements of M_1 are extremely lengthy for us to write it here. However, for some special choice of the parameters they take simple form. Note that the exponential terms in the mass matrix are $e^{C_b {x_0}^b}$ and $e^{\bar{C}_b \bar{x_0}^b}$, where ${x_0}^a = i p^a t_0$. Let us assume that $C_1 = C_2 = C_3 = C$ then the exponential terms take the form $e^{3 i t_0 C}$ and $e^{-3 i t_0 \bar{C}}$. We further assume that $C = ir$, r is real. The correction to the eigenvalues corresponding to the zero-modes of the mass matrix M_0 can be found by diagonalizing the matrix:

$$\langle\psi_i|M_1|\psi_j\rangle = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} . \tag{4.46}$$

The eigenvalues of this matrix are given by

$$\lambda_1 = -\frac{36\Im(A)e^{-3t_0r}(t_0r(t_0r(3t_0r+5)-5)-1)}{t_0^3} \tag{4.47}$$

$$\lambda_2 = -\frac{36\Im(A)e^{-3t_0r}(t_0r(t_0r(3t_0r+5)-5)-1)}{t_0^3} . \tag{4.48}$$

If we restrict $A = -ih$ to be negative imaginary, $h > 0$. For all $t_0r > 1$ we get positive values for both λ_1 and λ_2:

$$\lambda_1 = \frac{36he^{-3t_0r}(t_0r(t_0r(3t_0r+5)-5)-1)}{t_0^3} \tag{4.49}$$

$$\lambda_2 = \frac{36he^{-3t_0r}(t_0r(t_0r(3t_0r+5)-5)-1)}{t_0^3} \tag{4.50}$$

We can also diagonalize the full mass matrix numerically by assuming certain values for the parameters in the mass matrix. We take the values $A = -i$,$t_0 = 1$. Fig.1 below shows the variation of the six eigenvalues as a function of r. The last two plots correspond to the lift of both the zero-modes.

4.7 Exact result

It is in fact possible to find the eigenvalues of the mass matrix exactly for some special cases. For example, if we assume $C_1 = C_2 = C_3 = C$ and take C to be purely imaginary, we can diagonalize the mass matrix for generic values of A. However the solution extremely lengthy and we will not give it here. The solution becomes much simpler if we assume that $A = ih$, where h is real. The

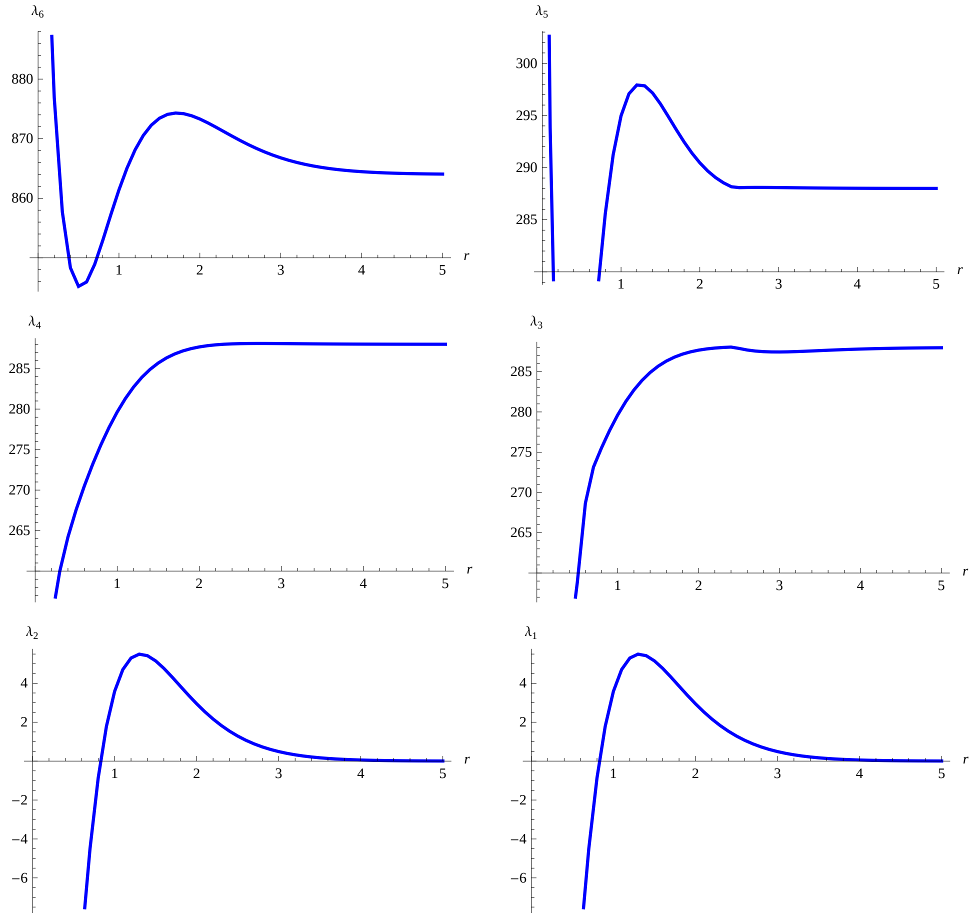

Figure 1: Variation of the 6 eigenvalues with r , r varies from 0 to 5

eigenvalues of the mass matrix for this case are listed below:

$$\lambda_1 = -36e^{-3t_0 r}\frac{h}{t_0}\Big(3t_0^3r^3 + 5t_0^2r^2 - 5t_0r - 1\Big) , \tag{4.51}$$

$$\lambda_1 = -36e^{-3t_0 r}\frac{h}{t_0}\Big(3t_0^3r^3 + 5t_0^2r^2 - 5t_0r - 1\Big) , \tag{4.52}$$

$$\lambda_3 = 288t_0^2 - 12e^{-3t_0 r}\frac{h}{t_0}\Big(9t_0^3r^3 - 21t_0^2r^2 + 3t_0r - 5\Big) , \tag{4.53}$$

$$\lambda_4 = 288t_0^2 - 12e^{-3t_0 r}\frac{h}{t_0}\Big(9t_0^3r^3 - 21t_0^2r^2 + 3t_0r - 5\Big) , \tag{4.54}$$

$$\lambda_5 = 288t_0^2 + 4e^{-3t_0 r}\frac{h}{t_0}\Big(81t_0^4r^4 - 189t_0^3r^3 - 99t_0^2r^2 + 192t_0r - 20\Big) , \tag{4.55}$$

$$\lambda_6 = 864t_0^2 - 4e^{-3t_0 r}\frac{h}{t_0}\Big(81t_0^4r^4 - 27t_0^3r^3 + 9t_0^2r^2 - 102t_0r + 26\Big) . \tag{4.56}$$

The first two eigenvalues λ_1 and λ_2 correspond to the zero-modes in the absence of instanton terms. The remaining four eigenvalues are corrected by exponentially suppressed terms due to the instanton correction. It is clear from the above expressions that by suitably adjusting the parameters the zero-modes get lifted up.

5 Conclusion

In this paper we have studied supersymmetric as well as non-supersymmetric attractors in the presence of sub-leading terms in the prepotential of the $N = 2$ supergravity theory. As a toy model, we considered the example of a two-parameter Calabi-Yau model where the prepotential is computed exactly to all orders using mirror symmetry. In this example, we considered single centered $D0 - D4$ black hole solutions. Interestingly, we observed that in the presence of sub-leading terms, the $D0-D4$ black hole is generically destabilized. Stable supersymmetric solutions exists when the $D4$-charges are restricted to the condition $p^2 = -2p^1$. The behavior of the non-supersymmetric solution is quite similar to the supersymmetric one. However, in this case, the massless direction still survives the perturbative corrections to the prepotential. In order to lift the massless mode, we considered instanton correction to the prepotential. We observed that, for the case of a three parameter model, the massless modes can in fact be lifted by the non-perturbative correction.

Throughout the paper, we have focused our attention to single centered $D0-D4$ black hole configurations. The $D0-D4-D6$ black holes behave pretty much the same way as the $D0 - D4$ system. Especially, for the non-supersymmetric attractors, the number of flat directions in the leading term of the effective black hole potential remains unchanged. It would be interesting to study the effect of sub-leading terms for the $D0 - D4 - D6$ system. It would also be interesting to study the walls of marginal stability in more detail. The condition on the charges in the presence of the sub-leading terms in the prepotential might get modified in the presence of curvature correction to the leading order $N = 2$ supergravity action. We hope to report on some of these issues in future.

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A Attractors in the two-parameter model

A.1 Effective Potential

In this section we give some details about the computations in section 3. We can calculate the Kähler metric using the equation $g_{a\bar{b}} = \partial_a \partial_{\bar{b}} K$ and its components are given by

$$\begin{aligned} g_{1\bar{1}} &= -6\frac{(2(x_1-\bar{x_1})^2 + 4(x_1-\bar{x_1})(x_2-\bar{x_2}) + 3(x_2-\bar{x_2})^2)}{(x_1-\bar{x_1})^2(2(x_1-\bar{x_1}) + 3(x_2-\bar{x_2}))} && \text{(A.1)} \\ g_{1\bar{2}} &= -\frac{6}{(2(x_1-\bar{x_1}) + 3(x_2-\bar{x_2}))^2} && \text{(A.2)} \\ g_{2\bar{1}} &= -\frac{6}{(2(x_1-\bar{x_1}) + 3(x_2-\bar{x_2}))^2} && \text{(A.3)} \\ g_{2\bar{2}} &= -\frac{9}{(2(x_1-\bar{x_1}) + 3(x_2-\bar{x_2}))^2} && \text{(A.4)} \end{aligned}$$

Inverse of $g_{a\bar{b}}$ is $g^{a\bar{b}}$ such that $g_{a\bar{b}} g^{\bar{b}c} = \delta_a{}^c$ and the components of $g^{a\bar{b}}$ are as follows

$$\begin{aligned} g^{1\bar{1}} &= -\frac{(x_1-\bar{x_1})^2}{2} && \text{(A.5)} \\ g^{1\bar{2}} &= \frac{(x_1-\bar{x_1})^2}{3} && \text{(A.6)} \\ g^{2\bar{1}} &= \frac{(x_1-\bar{x_1})^2}{3} && \text{(A.7)} \\ g^{2\bar{2}} &= -\frac{1}{3}\left(2(x_1-\bar{x_1})^2 + 4(x_1-\bar{x_1})(x_2-\bar{x_2}) + 3(x_2-\bar{x_2})^2\right) && \text{(A.8)} \end{aligned}$$

The covariant derivatives of the superpotential $\nabla_i W = \partial_i W + \partial_i K W$ reads

$$\begin{aligned}
\nabla_1 W &= \Big[\Big(2p^2 + 4p^2 x_1 + 4p^1(2x_1 + x_2)\Big)\Big(x_1 - \bar{x_1}\Big)\Big(2(x_1 - \bar{x_1}) + 3(x_2 - \bar{x_2})\Big) \\
&- \Big(6(p^2 + q_0 + 2p^2 x_1 + 2p^2 {x_1}^2) + 2p^1(11 + 12{x_1}^2 + 6x_2 + 12x_1 x_2)\Big) \\
&\quad \Big(x_1 - \bar{x_1} + x_2 - \bar{x_2}\Big)\Big]\Big[\Big(x_1 - \bar{x_1}\Big)\Big(2(x_1 - \bar{x_1}) + 3(x_2 - \bar{x_2})\Big)\Big]^{-1} \qquad \text{(A.9)} \\
\nabla_2 W &= \Big[\Big(p^1(2 + 4x_1)\Big)\Big(2(x_1 - \bar{x_1}) + 3(x_2 - \bar{x_2})\Big) - 3\Big(p^2 + q_0 + 2p^2 x_1 + 2p^2 {x_1}^2 \\
&+ p^1(11/3 + 4{x_1}^2 + 2x_2 + 4x_1 x_2)\Big)\Big]\Big[\Big(2(x_1 - \bar{x_1}) + 3(x_2 - \bar{x_2})\Big)\Big]^{-1} \qquad \text{(A.10)}
\end{aligned}$$

The effective black hole potential can be evaluated using the formula

$$V = e^K \Big[g^{a\bar{b}} \nabla_a W \, (\nabla_b W)^* + |W|^2\Big] \qquad \text{(A.11)}$$

After a bit tedious computations we get

$$\begin{aligned}
V &= \Big[24p^1 q_0 x_1 \bar{x_1} + 12p^2 q_0 x_1 \bar{x_1} + 44p^1 q_0 + 12p^2 q_0 + 64(p^1)^2 x_1^2 {\bar{x_1}}^2 \\
&+ 24(p^2)^2 x_1^2 {\bar{x_1}}^2 + 64p^1 p^2 x_1^2 {\bar{x_1}}^2 + 72(p^1)^2 x_1 x_2 \bar{x_1} \bar{x_2} + 96(p^1)^2 x_1 \bar{x_1} \\
&+ 30(p^2)^2 x_1 \bar{x_1} + 60p^1 p^2 x_1 \bar{x_1} + 24(p^1)^2 x_2 \bar{x_2} + (242/3)(p^1)^2 + 6(p^2)^2 \\
&+ 44p^1 p^2 + 6{q_0}^2 + \Big(12p^1 q_0 x_2 \bar{x_1} + 12p^1 q_0 x_1^2 + 6p^2 q_0 x_1^2 + 12p^2 q_0 x_1 \\
&+ 12p^1 q_0 x_2 + 12p^1 q_0 x_1 x_2 + 8(p^1)^2 x_2 {\bar{x_1}}^3 + 32(p^1)^2 x_2 {\bar{x_1}}^2 + 12p^1 p^2 x_2 {\bar{x_1}}^2 \\
&+ 64(p^1)^2 x_1 x_2 {\bar{x_1}}^2 + 24p^1 p^2 x_1 x_2 {\bar{x_1}}^2 + 16(p^1)^2 x_1^3 \bar{x_1} + 16p^1 p^2 x_1^3 \bar{x_1} \\
&+ 24(p^2)^2 x_1^2 \bar{x_1} + 40p^1 p^2 x_1^2 \bar{x_1} + 52(p^1)^2 x_2 \bar{x_1} + 24(p^1)^2 x_1^2 x_2 \bar{x_1} \\
&+ 24p^1 p^2 x_1^2 x_2 \bar{x_1} + 24p^1 p^2 x_2 \bar{x_1} + 16(p^1)^2 x_1 x_2 \bar{x_1} + 48p^1 p^2 x_1 x_2 \bar{x_1} \\
&+ 12(p^1)^2 x_1^2 x_2 \bar{x_2} + 48(p^1)^2 x_1 x_2 \bar{x_2} + 8p^1 p^2 x_1^3 + 40(p^1)^2 x_1^2 + 9(p^2)^2 x_1^2 \\
&+ 38p^1 p^2 x_1^2 + 12(p^2)^2 x_1 + 44p^1 p^2 x_1 + 44(p^1)^2 x_2 + 12p^1 p^2 x_1^2 x_2 \\
&+ 12p^1 p^2 x_2 + 36(p^1)^2 x_1 x_2 + 24p^1 p^2 x_1 x_2 + C.C.\Big)\Big] \\
&\times \frac{-i}{(x_1 - \bar{x_1})^2 (2(x_1 - \bar{x_1}) + 3(x_2 - \bar{x_2}))} \, . \qquad \text{(A.12)}
\end{aligned}$$

The non-supersymmetric solution is given by the solution of the the set of equations

$$\partial_i V_{eff} = 0 \qquad \text{(A.13)}$$

We have two complex equations $\partial_1 V_{eff} = 0$ and $\partial_2 V_{eff} = 0$. After substituting the ansatz $x_1 = p^1 t$ and $x_2 = p^2 t$ these equations read,

$$\begin{aligned}
& 18p^2(p^2+q_0)^2 + 16(p^1)^7\bar{t}^2(7t^2+10|t|^2+\bar{t}^2) + 64(p^1)^6p^2\bar{t}^2(7t^2+10|t|^2+\bar{t}^2) \\
+ & 6p^1(p^2+q_0)\Big(25p^2+3q_0+3(p^2)^2(3t+5\bar{t})\Big) + 2(p^1)^4\Big(18(p^2)^3\bar{t}^2(7t^2+10|t|^2 \\
+ & \bar{t}^2) + 6q_0(t^2+6|t|^2+5\bar{t}^2) + 12(p^2)^2\bar{t}(13t^2+38|t|^2+9\bar{t}^2) + p^2(55t^2+378|t|^2 \\
+ & 299\bar{t}^2)\Big) + 4(p^1)^5\Big(\bar{t}^2(54+20p^2\bar{t}+21(p^2)^2\bar{t}^2) + 2|t|^2(34+46p^2\bar{t}+105(p^2)^2\bar{t}^2) \\
+ & t^2(10+32p^2\bar{t}+147(p^2)^2\bar{t}^2)\Big) - 2(p^1)^2\Big[66q_0 + 18(p^2)^3(t^2+8|t|^2+5\bar{t}^2) \\
+ & p^2\Big(187+6q_0(5t+7\bar{t})\Big) + 3(p^2)^2\Big(3q_0t^2+3\bar{t}(23+5q_0\bar{t})+t(43+18q_0\bar{t})\Big)\Big] \\
+ & (p^1)^3\Big[242+36(p^2)^3\bar{t}(5t^2+15|t|^2+4\bar{t}^2) + 3(p^2)^2(37t^2+262|t|^2+181\bar{t}^2) \\
+ & 2p^2\Big(15q_0t^2+10t(11+9q_0\bar{t})+\bar{t}(154+75q_0\bar{t})\Big)\Big] = 0
\end{aligned} \tag{A.14}$$

$$\begin{aligned}
& 18(p^2+q_0)^2 + 16(p^1)^6\bar{t}^2(7t^2+10|t|^2+\bar{t}^2) + 12p^1(p^2+q_0)\Big(11+3p^2(t+3\bar{t})\Big) \\
+ & 16(p^1)^5\bar{t}\Big(\bar{t}^2(4+3p^2\bar{t}) + 2|t|^2(-1+15p^2\bar{t}) + t^2(-2+21p^2\bar{t})\Big) \\
+ & 4(p^1)^4\Big(\bar{t}^2(56+60p^2\bar{t}+9(p^2)^2\bar{t}^2) + 3t^2(4+4p^2\bar{t}+21(p^2)^2\bar{t}^2) \\
+ & 2|t|^2(32+36p^2\bar{t}+45(p^2)^2\bar{t}^2)\Big) + 2(p^1)^3\Big(t^2(33p^2+6q_0+72(p^2)^2\bar{t}) \\
+ & \bar{t}(44+237p^2\bar{t}+30q_0\bar{t}+108(p^2)^2\bar{t}^2) + 2t(-22+99p^2\bar{t}+18q_0\bar{t}+126(p^2)^2\bar{t}^2)\Big) \\
+ & (p^1)^2\Big[242-24q_0t+24q_0\bar{t}+9(p^2)^2(3t^2+26|t|^2+27\bar{t}^2) + 6p^2\Big(3q_0t^2 \\
+ & 18t(1+q_0\bar{t}) + 5\bar{t}(14+3q_0\bar{t})\Big)\Big] = 0
\end{aligned} \tag{A.15}$$

Substituting the condition $p^2 = -2p^1$ both the equations reduce to the following equation,

$$\begin{aligned}
30p^1q_0 + 9{q_0}^2 &+ 32(p^1)^6\bar{t}^2(7t^2+10|t|^2+\bar{t}^2) + 32(p^1)^5\bar{t}(7t^2+22|t|^2+7\bar{t}^2) \\
&+ 4(p^1)^4(3t^2+50|t|^2+31\bar{t}^2) + (p^1)^2(25-48q_0(t+2\bar{t})) - 4(p^1)^3(3q_0t^2 \\
&+ 5\bar{t}(8+3q_0\bar{t}) + 2t(10+9q_0\bar{t})) = 0\ .
\end{aligned} \tag{A.16}$$

Solving this equation, we find, for the non-supersymmetric black holes

$$t = -\frac{1}{2p^1} - \frac{i}{2\sqrt{2}p^1}\sqrt{-11-\frac{3q_0}{p^1}}\ . \tag{A.17}$$

A.2 Mass matrix

For the stability analysis of the attractor we need to calculate the mass matrix

$$M = \partial_b \partial_{\bar{a}} V \otimes \mathbf{I} + \Re(\partial_b \partial_a V) \otimes \sigma^{\mathbf{3}} - 2\Im(\partial_b \partial_a V) \otimes \sigma^{\mathbf{1}} \ . \tag{A.18}$$

For the mass matrix the second derivatives should be evaluated at the attractor point and we know that the solution exist only if we impose the condition $p^2 = -2p^1$. We have imposed this condition, evaluated the second derivatives and thereafter we have substituted the ansatz $x_1 = p^1 t$ and $x_2 = p^2 t$. Here we list the various second derivative terms.

$$\begin{aligned}
\partial_1 \partial_1 V &= \frac{3i}{4(p^1)^5(t-\bar{t})^5}\Big(30p^1 q_0 + 9{q_0}^2 + 32(p^1)^6 \bar{t}^2(7t^2 + 10t\bar{t} + \bar{t}^2) \\
&+ 32(p^1)^5 \bar{t}(7t^2 + 22t\bar{t} + 7\bar{t}^2) + 4(p^1)^4(3t^2 + 50t\bar{t} + 31\bar{t}^2) \\
&+ (p^1)^2(25 - 48q_0(t + 2\bar{t})) - 4(p^1)^3\Big(3q_0 t^2 + 5\bar{t}(8 + 3q_0\bar{t}) \\
&+ 2t(10 + 9q_0\bar{t})\Big)\Big)
\end{aligned} \tag{A.19}$$

$$\begin{aligned}
\partial_1 \partial_2 V &= \frac{i}{8(p^1)^5(t-\bar{t})^5}\Big(- 30p^1 q_0 - 9{q_0}^2 + (p^1)^2(-25 + 144q_0\bar{t}) \\
&+ 12(p^1)^4(t^2 - 2t\bar{t} - 27\bar{t}^2) - 32(p^1)^6\bar{t}^2(-7t^2 + 14t\bar{t} + 11\bar{t}^2) \\
&- 32(p1)^5\bar{t}(-7t^2 + 14t\bar{t} + 29\bar{t}^2) + 12(p^1)^3\Big(20\bar{t} + q_0(-t^2 \\
&+ 2t\bar{t} + 11\bar{t}^2)\Big)\Big)
\end{aligned} \tag{A.20}$$

$$\begin{aligned}
\partial_2 \partial_2 V &= \frac{3i}{16(p^1)^5(t-\bar{t})^5)}\Big(30p^1 q_0 + 9{q_0}^2 + 32(p^1)^6 \bar{t}^2(7t^2 + 10t\bar{t} + \bar{t}^2) \\
&+ 32(p^1)^5 \bar{t}(7t^2 + 22t\bar{t} + 7\bar{t}^2) + 4(p^1)^4(3t^2 + 50t\bar{t} + 31\bar{t}^2) \\
&+ (p^1)^2(25 - 48q_0(t + 2\bar{t})) - 4(p^1)^3\Big(3q_0 t^2 + 5\bar{t}(8 + 3q_0\bar{t}) \\
&+ 2t(10 + 9q_0\bar{t})\Big)\Big)
\end{aligned} \tag{A.21}$$

$$\begin{aligned}
\partial_1 \partial_{\bar{1}} V &= \frac{-3i}{4(p^1)^5(t-\bar{t})^5)}\Big(30p^1 q_0 + 9{q_0}^2 + 96(p^1)^6 t\bar{t}(t^2 + 4t\bar{t} + \bar{t}^2) \\
&+ 12(p^1)^4(3t^2 + 22t\bar{t} + 3\bar{t}^2) + 48(p^1)^5(t^3 + 11t^2\bar{t} + 11t\bar{t}^2 + \bar{t}^3) \\
&+ (p^1)^2(25 - 72q_0(t + \bar{t})) - 12(p^1)^3(t + \bar{t})(10 + 3q_0(t + \bar{t}))\Big)
\end{aligned} \tag{A.22}$$

$$
\begin{aligned}
\partial_1\partial_{\bar{2}}V &= \frac{i}{8(p^1)^5(t-\bar{t})^5)}\Big(30p^1q_0+9{q_0}^2+28(p^1)^4(5t^2+2t\bar{t}+5\bar{t}^2)\\
&+ 32(p^1)^6t\bar{t}(5t^2+8t\bar{t}+5\bar{t}^2)+16(p^1)^5(5t^3+31t^2\bar{t}+31t\bar{t}^2+5\bar{t}^3)\\
&+ (p^1)^2(25-72q_0(t+\bar{t}))-12(p^1)^3\Big(q_0t^2+10t(1+q_0\bar{t})\\
&+ \bar{t}(10+q_0\bar{t})\Big)\Big)
\end{aligned} \tag{A.23}
$$

$$
\begin{aligned}
\partial_2\partial_{\bar{2}}V &= \frac{-3i}{16(p^1)^5(t-\bar{t})^5)}\Big(30p^1q_0+9{q_0}^2+96(p^1)^6t\bar{t}(t^2+4t\bar{t}+\bar{t}^2)\\
&+ 12(p^1)^4(3t^2+22t\bar{t}+3\bar{t}^2)+48(p^1)^5(t^3+11t^2\bar{t}+11t\bar{t}^2+\bar{t}^3)\\
&+ (p^1)^2(25-72q_0(t+\bar{t}))-12(p^1)^3(t+\bar{t})(10+3q_0(t+\bar{t}))\Big)
\end{aligned} \tag{A.24}
$$

Upon substituting the solution $t=-\frac{1}{2p^1}-\frac{i}{2\sqrt{2}p^1}\sqrt{-11-\frac{3q_0}{p^1}}$ these terms take the form

$$\partial_1\partial_1V = 0 \tag{A.25}$$

$$\partial_1\partial_2V = \frac{-4\sqrt{2}(p^1)^2}{\sqrt{-11-\frac{3q_0}{p^1}}} \tag{A.26}$$

$$\partial_2\partial_2V = 0 \tag{A.27}$$

$$\partial_1\partial_{\bar{1}}V = \frac{12\sqrt{2}(p^1)^2}{\sqrt{-11-\frac{3q_0}{p^1}}} \tag{A.28}$$

$$\partial_1\partial_{\bar{2}}V = \frac{-2\sqrt{2}(p^1)^2}{\sqrt{-11-\frac{3q_0}{p^1}}} \tag{A.29}$$

$$\partial_2\partial_{\bar{2}}V = \frac{3\sqrt{2}(p^1)^2}{\sqrt{-11-\frac{3q_0}{p^1}}} \tag{A.30}$$

Substituting this terms in the expression for mass matrix we get

$$
M=\frac{\sqrt{2}(p^1)^2}{\sqrt{-11-\frac{3q_0}{p^1}}}\begin{pmatrix} 12 & 0 & -6 & 0\\ 0 & 12 & 0 & 2\\ -6 & 0 & 3 & 0\\ 0 & 2 & 0 & 3\end{pmatrix} \tag{A.31}
$$

B Instanton corrections

B.1 Non-supersymmetric solution

In this section we will work out the non-supersymmetric solution by brute force method. First we will obtain the non-supersymmetric solution by imposing $x^a = p^a t$ and assuming $t = m + i\,(t_0 + s)$ then we will solve for m and s from the equation

$$\partial_a V = e^K \left(\partial_a U + \partial_a K U\right) = 0 \tag{B.1}$$

We have

$$\begin{aligned}
\partial_a U &= \partial_a U_0 + \left(C_a U_1 + \partial_a U_1\right) A e^{C_b x^b} + \partial_a \bar{U}_1 \bar{A} e^{\bar{C}_b \bar{x}^b} && \text{(B.2)} \\
\partial_a K &= \frac{-1}{M+L}\Big[3M_a + C_a(k-1)Ae^{C_b x^b} + \bar{C}_a \bar{A} e^{\bar{C}_b \bar{x}^b}\Big] && \text{(B.3)}
\end{aligned}$$

Substituting these expressions in $\partial_a V$ we get

$$\begin{aligned}
e^{-K}\partial_a V &= \frac{1}{M+L}\Big[M\partial_a U_0 - 3M_a U_0 + L\partial_a U_0 \\
&+ \Big(MC_a U_1 + M\partial_a U_1 - C_a(k-1)U_0 - 3M_a U_1\Big)Ae^{C_b x^b} \\
&+ \Big(M\partial_a U_2 - \bar{C}_a U_0 - 3M_a U_2\Big)\bar{A}e^{\bar{C}_b \bar{x}^b}\Big] && \text{(B.4)}
\end{aligned}$$

It is convenient to solve $p^a \partial_a V = 0$, so we contract $\partial_a V$ with p^a and substitute for L we get

$$\begin{aligned}
e^{-K} p^a \partial_a V &= \frac{p^a}{M+L}\Big[M\partial_a U_0 - 3M_a U_0 + \Big(\partial_a U_0(k-2) + MC_a U_1 + M\partial_a U_1 \\
&- C_a(k-1)U_0 - 3M_a U_1\Big)Ae^{C_b x^b} + \Big(-\partial_a U_0(\bar{k}-2) + M\partial_a \bar{U}_1 \\
&- \bar{C}_a U_0 - 3M_a \bar{U}_1\Big)\bar{A}e^{\bar{C}_b \bar{x}^b}\Big] && \text{(B.5)}
\end{aligned}$$

We have calculated all the terms in $e^{-K}p^a\partial_a V$ and they are as follows,

$$\begin{aligned}
p^a(M\partial_a U_0 - 3M_a U_0) &= -192iD^3{t_0}^5\,(3m - is)) && \text{(B.6)}\\
p^a\partial_a U_0(k-2) &= -48iD^2{t_0}^3(iTt_0 - 1) && \text{(B.7)}\\
p^a M C_a U_1 &= -16iD^2T{t_0}^4(-3i - Tt_0 + iT^2{t_0}^2) && \text{(B.8)}\\
p^a M\partial_a U_1 &= -8iD^2{t_0}^3(4iTt_o + 3T^2{t_0}^2 - 3) && \text{(B.9)}\\
-p^a C_a(k-1)U_0 &= -16D^2T{t_0}^4(2iTt_0 - 1) && \text{(B.10)}\\
-3p^a M_a U_1 &= 24D^2{t_0}^3(-3i - Tt_0 + iT^2{t_0}^2) && \text{(B.11)}\\
-p^a\partial_a U_0(\bar{k}-2) &= -48iD^2{t_0}^3(i\bar{T}t_0 + 1) && \text{(B.12)}\\
p^a M\partial_a \bar{U}_1 &= -8iD^2{t_0}^3(8i\bar{T}t_0 + \bar{T}^2{t_0}^2 + 3) && \text{(B.13)}\\
-p^a\bar{C}_a U_0 &= -16D^2\bar{T}{t_0}^4 && \text{(B.14)}\\
-3p^a M_a\bar{U}_1 &= 24D^2{t_0}^3(3i - \bar{T}t_0 - i\bar{T}^2{t_0}^2) && \text{(B.15)}
\end{aligned}$$

Where $t_0 = \sqrt{\frac{-q_0}{D}}$. Substituting the above terms we get

$$\begin{aligned}
e^{-K}p^a\partial_a V &= \frac{1}{M+L}\Big[-192iD^3{t_0}^5(3m - is) + 8D^2{t_0}^4\Big(3T - 2it_0T^2\\
&\quad +2\ \ {t_0}^2T^3\Big)Ae^{C_b{x_0}^b} + 8D^2{t_0}^4\Big(9\bar{T} - 4it_0\bar{T}^2\Big)\bar{A}e^{\bar{C}_b\bar{x_0}^b}\Big] \qquad \text{(B.16)}
\end{aligned}$$

$\partial_\alpha V = 0$ implies that

$$3m - is = \frac{1}{24iDt_0}\Big[\Big(3T - 2it_0T^2 + 2{t_0}^2T^3\Big)Ae^{C_b{x_0}^b} + \Big(9\bar{T} - 4it_0\bar{T}^2\Big)\bar{A}e^{\bar{C}_b\bar{x_0}^b}\Big] \qquad \text{(B.17)}$$

Solving for m and s we get

$$m = \frac{1}{36Dt_0}\Im\Big(T\big(-3 - 3it_0T + T^2{t_0}^2\big)Ae^{C_b{x_0}^b}\Big) \qquad \text{(B.18)}$$

and

$$s = -\frac{1}{12Dt_0}\Re\Big(T\big(6 + it_0T + {t_0}^2T^2\big)Ae^{C_b{x_0}^b}\Big)\ . \qquad \text{(B.19)}$$

For the above solution, we find

$$\begin{aligned}
U|_{\phi_{i0}} &= 16D^2{t_0}^4 - Dt_0\Re\Big((-3i + 7Tt_0 + {t_0}^3T^3)Ae^{C_b{x_0}^b}\Big)\ , && \text{(B.20)}\\
e^K|_{\phi_{i0}} &= \frac{1}{8D{t_0}^3} + \frac{1}{32D^2{t_0}^6}\Re\Big((2i + 8Tt_0 + i{t_0}^2T^2 + {t_0}^3T^3)Ae^{C_b{x_0}^b}\Big)\ . && \text{(B.21)}
\end{aligned}$$

Hence, the entropy of the black hole is given by

$$S = 2\pi Dt_0 + \frac{\pi}{2{t_0}^2}\Re\Big(\big(5i + 7Tt_0 + i{t_0}^2T^2\big)Ae^{C_b{x_0}^b}\Big)\ . \qquad \text{(B.22)}$$

B.2 Mass matrix

The matrix elements are given by the second derivative of the effective potential. The required terms are $e^{-K_0}\partial_b\partial_a V$ and $e^{-K_0}\partial_{\bar{b}}\partial_a V$.

$$\partial_b\partial_a V = \Big(\partial_b\partial_a U + \partial_b\partial_a K U + \partial_a K \partial_b U\Big)e^K \ . \tag{B.23}$$

Using the expression for U we can write

$$\begin{aligned} \partial_b\partial_a U &= \partial_b\partial_a U_0 + Ae^{c_b x_0{}^b}\Big(\partial_b\partial_a U_1 + C_a C_b U_1 + C_b\partial_a U_1 + C_a\partial_b U_1\Big) \\ &+ \bar{A}e^{\bar{c}_b\bar{x}_0^b}\Big(\partial_b\partial_a\bar{U}_1\Big) \end{aligned} \tag{B.24}$$

Using the equation of motion we can rewrite

$$\partial_b\partial_a K + \partial_a K\partial_b \ = \ \partial_b\partial_a K - \partial_a K\partial_b K \tag{B.25}$$

After doing little bit algebra we can conclude,

$$\partial_b\partial_a K - \partial_a K\partial_b K \ = \ \frac{-1}{M+L}\Big(6M_{ab} + kC_a C_b Ae^{C_b x^b}\Big) \tag{B.26}$$

$$= \ H_0 + H_1 Ae^{C_b x^b} \tag{B.27}$$

Using this expression we can write

$$(\partial_b\partial_a K - \partial_a K\partial_b K)U \ = \ H_0 U_0 + (H_1 U_0 + H_0 U_1)Ae^{C_b x_0{}^b} + H_0\bar{U}_1\bar{A}e^{\bar{C}_b\bar{x}^b} \tag{B.28}$$

Using the above equations we can write

$$\begin{aligned} \partial_b\partial_a V e^{-K} &= \partial_b\partial_a U_0 + H_0 U_0 + Ae^{c_b x_0{}^b}\Big(\partial_b\partial_a U_1 + C_a C_b U_1 \\ &+ C_b\partial_a U_1 + C_a\partial_b U_1 + H_0 U_1 + H_1 U_0\Big) \\ &+ \bar{A}e^{\bar{c}_b\bar{x}_0^b}\Big(\partial_b\partial_a U_2 + H_0\bar{U}_1\Big) \end{aligned} \tag{B.29}$$

We have evaluated each term in the above expression at the critical value and their values are listed below,

$$\begin{aligned}
\partial_b\partial_a U_0 &= D_{ab}\Big[T\Big(10+5iTt_0-\frac{1}{3}T^2{t_0}^2\Big)Ae^{c_b{x_0}^b} \\
&+ \bar{T}\Big(2+3i\bar{T}t_0+\frac{7}{3}\bar{T}^2{t_0}^2\Big)\bar{A}e^{\bar{c}_b\bar{x}_0^b}\Big] && \text{(B.30)}\\
H_0U_0 &= 24D{t_0}^2D_{ab}+\frac{2D_{ab}}{t_0}\Re\Big[\Big(6i-i{t_0}^2T^2-{t_0}^3T^3\Big)Ae^{c_b{x_0}^b}\Big] && \text{(B.31)}\\
\partial_b\partial_a U_1 &= \frac{3D_{ab}}{t_0}\Big(-i+5t_0T+3i{t_0}^2T^2\Big)+\frac{9D_aD_b}{Dt_0}\Big(i-3Tt_0-i{t_0}^2T^2\Big)\\
&+ 2J_{ab}\Big(D+iDTt_0\Big)+\frac{3}{2}\Big(D_aC_b+D_bC_a\Big)\Big(7+2iTt_0\Big)\\
&- 9iDt_0C_aC_b && \text{(B.32)}\\
C_aC_bU_1 &= 2Dt_0C_aC_b\Big(-3i-Tt_0+iT^2{t_0}^2\Big) && \text{(B.33)}\\
C_a\partial_b U_1 &= Dt_0\Big(-5i+6Tt_0\Big)C_aC_b+3\Big(-1+3iTt_0-T^2{t_0}^2\Big)D_bC_a && \text{(B.34)}\\
C_b\partial_a U_1 &= Dt_0\Big(-5i+6Tt_0\Big)C_aC_b+3\Big(-1+3iTt_0-T^2{t_0}^2\Big)D_aC_b && \text{(B.35)}\\
H_0U_1 &= (-3i+t_0T+i{t_0}^2T^2)\frac{3D_{ab}}{t_0} && \text{(B.36)}\\
H_1U_0 &= 4TD{t_0}^2C_aC_b && \text{(B.37)}
\end{aligned}$$

$$\begin{aligned}
\partial_a\partial_b\bar{U}_1 &= 3\Big(i+5\bar{T}t_0-3i\bar{T}^2{t_0}^2\Big)\frac{D_{ab}}{t_0}+\frac{9D_aD_b}{Dt_0}\Big(-i+\bar{T}t_0+i\bar{T}^2{t_0}^2\Big)\\
&+ 2D\Big(-1+i\bar{T}t_0\Big)\bar{J}_{ab}-\frac{3}{2}\Big(1+2it_0\bar{T}\Big)\Big(D_a\bar{C}_b+D_b\bar{C}_a\Big)\\
&+ iDt_0\bar{C}_a\bar{C}_b && \text{(B.38)}\\
H_0U_2 &= (3i-\bar{T}t_0-i\bar{T}^2{t_0}^2)\frac{3D_{ab}}{t_0} && \text{(B.39)}
\end{aligned}$$

Adding up the above results we get

$$
\begin{aligned}
e^{-K}\partial_b\partial_a V &= 24DD_{ab}{t_0}^2 + \Big[\frac{2D_{ab}}{t_0}\Big(-3i + 11t_0T + 8iT^2{t_0}^2 - \frac{2}{3}T^3{t_0}^3\Big) \\
&+ \frac{9D_aD_b}{Dt_0}\Big(i - 3t_0T - iT^2{t_0}^2\Big) + C_aC_bDt_0\Big(-25i + 14Tt_0 \\
&+ 2iT^2{t_0}^2\Big) + 2D\Big(1 + it_0T\Big)J_{ab} + \frac{3}{2}\Big(5 - 2T^2{t_0}^2 + 8iTt_0\Big) \\
&\Big(D_aC_b + D_bC_a\Big)\Big]Ae^{c_bx_0{}^b} + \Big[2\Big(3i + 7\bar{T}t_0 - 4i\bar{T}^2{t_0}^2 \\
&+ \frac{2}{3}\bar{T}^3{t_0}^3\Big)\frac{D_{ab}}{t_0} + \frac{9D_aD_b}{Dt_0}\Big(-i + \bar{T}t_0 + i\bar{T}^2{t_0}^2\Big) \\
&+ 2D\Big(-1 + i\bar{T}t_0\Big)\bar{J}_{ab} + iDt_0\bar{C}_a\bar{C}_b - \frac{3}{2}\Big(1 + i\bar{T}t_0\Big) \\
&\Big(D_a\bar{C}_b + D_b\bar{C}_a\Big)\Big]\bar{A}e^{\bar{c}_b\bar{x}_0^b} \qquad \text{(B.40)}
\end{aligned}
$$

Consider the term $\partial_{\bar{b}}\partial_a V$ in the mass matrix. Using the equations of motion we can write it as

$$\partial_{\bar{b}}\partial_a V = e^K\Big(\partial_{\bar{b}}\partial_a KU + \partial_{\bar{b}}\partial_a U - (\partial_a K)(\partial_{\bar{b}}K)U\Big) \qquad \text{(B.41)}$$

Using the expression for U we can write,

$$\partial_{\bar{b}}\partial_a U = \partial_{\bar{b}}\partial_a U_0 + Ae^{c_bx_0{}^b}\left(\partial_{\bar{b}}\partial_a U_1 + C_a\partial_{\bar{b}}U_1\right) + \bar{A}e^{\bar{C}_b\bar{x}_0^b}\left(\partial_{\bar{b}}\partial_a\bar{U}_1 + \bar{C}_b\partial_a\bar{U}_1\right) \qquad \text{(B.42)}$$

After doing little bit algebra it can be shown that

$$
\begin{aligned}
\partial_{\bar{b}}\partial_a K - \partial_a K\partial_{\bar{b}}K &= \frac{1}{M+L}\Big(6M_{ab} + C_aC_bAe^{C_bx^b} \\
&- \bar{C}_a\bar{C}_b\bar{A}e^{\bar{C}_b\bar{x^b}}\Big) \\
&= G_0 + G_1Ae^{C_bx^b} + G_2\bar{A}e^{\bar{C}_b\bar{x^b}} \qquad \text{(B.43)} \\
& \qquad \text{(B.44)}
\end{aligned}
$$

We have introduced G_0, G_1 and G_2 for notational simplicity. Multiplying with U gives

$$(\partial_{\bar{b}}\partial_a K - \partial_a K\partial_{\bar{b}}K)U = G_0U_0 + (G_1U_0 + G_0U_1)Ae^{C_bx^b} + (G_2U_0 + G_0U_2)\bar{A}e^{\bar{C}_b\bar{x^b}} \qquad \text{(B.45)}$$

Substituting for each terms in $\partial_{\bar{b}}\partial_a V$ we get

$$
\begin{aligned}
e^{-K}\partial_{\bar{b}}\partial_a V &= \partial_{\bar{b}}\partial_a U_0 + G_0 U_0 + A e^{c_b x_0{}^b}\Big(G_1 U_0 + G_0 U_1 + \partial_{\bar{b}}\partial_a U_1 + C_a \partial_{\bar{b}} U_1\Big) \\
&+ \bar{A} e^{\bar{c}_b \bar{x}_0{}^b}\Big(G_0 \bar{U}_1 + G_2 U_0 + \partial_{\bar{b}}\partial_a \bar{U}_1 + \bar{C}_b \partial_a \bar{U}_1\Big)
\end{aligned}
\tag{B.46}
$$

We have evaluated each term in the above expression explicitly at the critical value and they are listed below.

$$
U_0 G_0 = -24 D t_0{}^2 D_{ab} + \frac{2D_{ab}}{t_0}\Re\Big[\Big(-6i + i t_0{}^2 T^2 + t_0{}^3 T^3\Big) A e^{C_b x^b}\Big] \tag{B.47}
$$

$$
\begin{aligned}
\partial_{\bar{b}}\partial_a U_0 &= 72 t_0{}^2 D_a D_b \\
&+ 2T\Big(D_{ab} - \frac{6 D_a D_b}{D}\Big)\Re\Big[\Big(6 + iTt_0 + T^2 t_0{}^2\Big) e^{C_b x_0{}^b}\Big]
\end{aligned}
\tag{B.48}
$$

$$
G_1 U_0 = 2iDt_0 C_a C_b \tag{B.49}
$$

$$
G_0 U_1 = \frac{-3D_{ab}}{t_0}\Big(-3i - Tt_0 + iT^2 t_0{}^2\Big) \tag{B.50}
$$

$$
\begin{aligned}
\partial_{\bar{b}}\partial_a U_1 &= 3\Big(7i + Tt_0 - iT^2 t_0{}^2\Big)\frac{D_{ab}}{t_0} + \Big(-i - 7Tt_0 + 9iT^2 t_0{}^2\Big)\frac{D_a D_b}{D t_0} \\
&+ 2iDt_0 T J_{ab} + 5iDt_0 C_a C_b - \frac{3}{2}\Big(19 + 2iTt_0\Big) D_a C_b \\
&+ \frac{3}{2}\Big(5 - 2iTt_0\Big) D_b C_a
\end{aligned}
\tag{B.51}
$$

$$
C_a \partial_{\bar{b}} U_1 = Dt_0\Big(i - 2Tt_0\Big) C_a C_b + 3\Big(1 - 3iTt_0 + T^2 t_0{}^2\Big) D_b C_a \tag{B.52}
$$

$$
U_0 G_2 = -2iDt_0 \bar{C}_a \bar{C}_a \tag{B.53}
$$

$$
G_0 \bar{U}_1 = \frac{-3D_{ab}}{t_0}\Big(3i - \bar{T}t_0 - i\bar{T}^2 t_0{}^2\Big) \tag{B.54}
$$

$$
\begin{aligned}
\partial_{\bar{c}}\partial_a \bar{U}_1 &= \frac{3D_{ab}}{t_0}\Big(-7i + \bar{T}t_0 + i\bar{T}^2 t_0{}^2\Big) + \frac{D_a D_b}{D t_0}\Big(i - 7\bar{T}t_0 - 9i\bar{T}^2 t_0{}^2\Big) \\
&- 2iD\bar{T}t_0 \bar{J}_{ab} - 5iDt_0 \bar{C}_a \bar{C}_b + \frac{3}{2}\Big(5 + 2i\bar{T}t_0\Big) D_a \bar{C}_b \\
&+ \frac{3}{2}\Big(-19 + 2i\bar{T}t_0) D_b \bar{C}_a
\end{aligned}
\tag{B.55}
$$

$$
\bar{C}_b \partial_a U_2 = -Dt_0\Big(i + 2\bar{T}t_0\Big)\bar{C}_a \bar{C}_b + 3\Big(1 + 3it_0\bar{T} + \bar{T}^2 t_0{}^2\Big) D_a \bar{C}_b \tag{B.56}
$$

Adding up the terms we get

$$
\begin{aligned}
e^{-K}\partial_{\bar{b}}\partial_a V &= 24D{t_0}^2\Big(\frac{3D_aD_b}{D} - D_{ab}\Big) \\
&+ \Big[Ae^{c_b{x_0}^b}\Big(\frac{2D_{ab}}{t_0}\Big(12i + 6t_0T - 2i{t_0}^2T^2 + T^3{t_0}^3\Big) \\
&+ \frac{D_aD_b}{Dt_0}\Big(-i - 43t_0T + 3iT^2{t_0}^2 - 6T^3{t_0}^3\Big) \\
&+ 2Dt_0C_aC_b\Big(4i - 2Tt_0\Big) + 2iDt_0TJ_{ab} - \frac{3}{2}D_aC_b\Big(19 + 2iTt_0\Big) \\
&+ \frac{3}{2}D_bC_a\Big(7 - 8it_0T + 2T^2{t_0}^2\Big)\Big) + H.C.\Big] \qquad \text{(B.57)}
\end{aligned}
$$

The real and imaginary part of $\Re(\partial_b\partial_a V)$, $\Im(\partial_b\partial_a V)$ are given by

$$
\begin{aligned}
\Re(\partial_b\partial_a V) &= 24DD_{ab}{t_0}^2 + \Big[Ae^{c_b{x_0}^b}\Big(\frac{6D_{ab}}{t_0}(-i + 3Tt_0 + 2iT^2{t_0}^2\Big) \\
&+ \frac{9D_aD_b}{Dt_0}\Big(i - Tt_0 - iT^2{t_0}^2\Big) \\
&+ Dt_0\Big(-13i + 7Tt_0 + iT^2{t_0}^2\Big)C_aC_b \\
&+ \frac{3}{2}\Big(2 + 5it_0T - T^2{t_0}^2\Big)\Big(D_aC_b + D_bC_a)\Big) + C.C.\Big] \qquad \text{(B.58)} \\
\Im(\partial_b\partial_a V) &= \Big[Ae^{c_b{x_0}^b}\Big(\frac{4TD_{ab}}{t_0}\Big(-i + Tt_0 + \frac{1}{3}iT^2{t_0}^2\Big) \\
&+ 18it_0T\frac{D_aD_b}{Dt_0} + 2D\Big(-i + t_0T\Big)J_{ab} \\
&+ Dt_0\Big(-12 - 7iTt_0 + T^2t_0\Big)C_aC_b \\
&+ \frac{3}{2}\Big(-3i + iT^2{t_0}^2 + 3Tt_0\Big)\Big(D_aC_b + D_bC_a\Big)\Big) + C.C.\Big] \qquad \text{(B.59)}
\end{aligned}
$$

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